Calculus 251:C3 Reading Guide - 6/14/2021

Section 14.5 Directional Derivatives and Gradient Vectors

NOTE1: This material is NOT going to be on midterm 1!

NOTE 2: We are finally at the part of the course where we only do one section of the book today!

In this section, we are going to learn about the gradient, one of the most useful concepts in all of multivariable calculus. By definition, it looks like we are just giving a name to the vector whose components are the partial derivatives of a function. But this vector has some amazing properties which will allow use to find directional derivatives, paths of steepest ascent, and normal vectors to level curves and surfaces. The gradient will also be a critical tool that we use to analyze vector fields in chapter 16.

One note about directional derivatives: they are really just partial derivatives in some direction which may not be parallel to a coordinate axis. Another way to think about this is that f_x is really the same thing as $D_{\mathbf{i}}f$, the directional derivative in the direction of the positive x-axis.

Find the following definitions/concepts/formulas/theorems:

- directional derivative (definition and formula for finding it using the dot product)
- gradient
- properties of the directional derivative
- tangent line to a level curve - Big question: what would you get if you extended this formula into $\mathbb{R}^3$? What would the formula be, and what would it represent? Could you keep going into higher dimensions?
- Algebra rules for gradients
- extension of gradient into three dimensions
- Chain Rule for Paths

The motivation for directional derivatives (all of the stuff above the definition) is useful for visualizing what's going on. Please read it.

Example 1 is a direct calculation using the limit definition of the directional derivative. Please don't ever calculate a directional derivative this way if you can avoid it. Example 2 is how you should be calculating directional derivatives.

Example 3 illustrates how the gradient relates to the direction of greatest increase/descrease and the directions of zero change (which are tangent to the level curve or surface as one would expect). This type of question is certainly fair game for homework, quiz, and exam (not exam 1!) questions.

Example 4 shows how to use the gradient to find the tangent line to an implicitly defined curve in $\mathbb{R}^2$. What we are really doing in this example is "going up a dimension" to think about this 2D object as a level curve of some 3D object. We certainly could use calculus 1 implicit differentiation techniques to find this tangent line, but the calculations are much faster and cleaner this way.

If you are curious, you should try proving one or more of the properties of the algebra rules for gradients from the definitions.

Example 5 applies the algebra rules in a couple of typical ways. There should be no surprises here. Example 6 is a straightforward application of the gradient for a function of three variables. Directly above example 6, there is a sentence that says, "In any direction orthogonal to ∇f, the derivative is zero." How many such directions are there? If you put all of those directions together, what shape do you get?