Calculus 251:C3 Reading Guide - 6/7/2021

Section 13.1 Vector-Valued Functions and motion in space

You can think of vector-valued functions as functions which take a real number as input and return a vector as output. We have already seen some vector-valued functions, specifically the parametrizations of lines. This section will extend our view of vector-valued functions to include general curves in $\mathbb{R}^3$. There are many useful pictures of curves and surfaces in $\mathbb{R}^3$, both in the body of the section and in the exercises. I won't be able to draw any of these, so enjoy them in the textbook!

Find the following definitions/concepts:

- curve
- path
- What is the difference between the path and the curve traced by a vector-valued function?
- component functions
- vector-valued function
- scalar function
- limit (of a vector-valued function)
- continuous (for a vector-valued function)
- formula for derivative of a vector-valued function
- smooth, piecewise smooth
- tangent vector (a/k/a velocity vector)
- tangent line (as it applies to vector-valued functions)
- direction of motion/speed
- acceleration vector
- differentiation rules for vector-valued functions (Why are there multiple product rules?)

Example 1 is a nice visual way to think about how we would describe a helix. Note that in the parametrization, the first two components would describe a circle if the third component were constant. But the third component is linear in t (this time, it is just t), which produces motion in the z-direction. When you see a parametrization that looks like this, you should recognize that you have a helix. What would happen to the graph if we switched the first and third components?

Examples 2 and 3 are examples of limits and continuity for vector-values functions.

Example 4 is a basic example of a motion problem in space. Please make sure you are comfortable with it.

The proofs of the Dot Product, Cross Product, and Chain Rules are useful if you intend to major in math or if you need to be convinced why these rules work the way they do. You can skip them if you are not interested.

The last part of this section discusses the (possibly surprising) result that if a vector-valued function is of constant magnitude, then it is orthogonal to its derivative. You may actually have seen this before in the context of centripetal forces, but probably only in two dimensions. The key visualization is that a constant-magnitude vector function represents movement on the surface of a sphere, and the velocity is always tangent to the surface of that sphere. Because the position vector is a radius of the sphere, the two vectors (position and velocity) must be orthogonal.

Section 13.2 Integrals of Vector Functions; Projectile Motion

The calculus of vector functions (this section and part of the previous section) can be summarized in the following two sentences. When you want the {limit, derivative, integral} of a vector-valued function, do everything component-wise. Pay attention to the order of the functions when you use the three different product rules, because it matters for one of them.

Find the following definitions/concepts/formulas/theorems:

- antiderivative, indefinite integral, and definite integral of a vector-valued function
- concept extension from single-variable: two antiderivatives of a vector-valued function differ by a constant
- Fundamental Theorem of Calculus: vector-valued function edition

Examples 1, 2, and 3 involve integration of vector functions. Example 3 is a classic example of the two-sentence summary above. In single-variable calculus, we gave you acceleration (in one dimension) as a function of time along with two initial conditions (typically the initial velocity and position). You then integrated twice, using one of the initial conditions to solve for each of the constants of integration. This example is really the exact same process, but you are solving three initial value problems in parallel, one for each component x, y, z.

The remainder of the section includes a bunch of formulas and two examples for projectile motion. These are nice formulas, and if you are going to do a lot of physics problems then memorizing them might be useful. Honestly, I usually just set up these problems without using any of the cool formulas and use my calculus skills to get the same answer. More algebra, less memorization. That's how I roll. You will find that there is often a tradeoff between memorizing additional formulas (or additional forms of the same formula) and doing extra algebra. As you have already heard me say and seen me do in class, I almost always do the extra algebra to save room in my brain for things I actually need to remember. Your mileage may vary.