Calculus 251:C3 Reading Guide - 6/3/2021

Section 12.5 Lines and Planes in Space

In this section, we are going to take many of the concepts that you are (hopefully) very comfortable with regarding lines in $\mathbb{R}^2$ and extend them to lines and planes in $\mathbb{R}^3$. Back in high school algebra, you learned that the standard form of the equation of a line is $ax + by = c$. We called this a linear equation because its graph in $\mathbb{R}^2$ is a line. But the concept of linear equations generalizes to any number of dimensions. In fact, any equation in which every variable appears with only a (possibly zero) coefficient is called a linear equation in any number of dimensions. Of course, in dimensions higher than two the graph is no longer a line. But recall the term "linear combination" as we applied it to vectors: this is the sense in which these equations are linear. These equations are all of the form "some linear combination of the variables=constant". In $\mathbb{R}^3$, every linear equation describes a plane.

Find the following definitions/formulas/theorems:

- Equation of a line in $\mathbb{R}^3$ (vector parameterization)
- Equation of a line in $\mathbb{R}^3$ (Parametric equations)
- normal vector (reminder from Calculus I: the normal line to a curve at a point is perpendicular to the tangent line at the same point)
- Geometric description of a plane
- Equations of a plane (3 different forms)
- When are two planes parallel?
- collinear points
- angle between planes
- distance from a point to a line
- distance from a point to a plane

Examples 1, 2, and 3 involve finding equations for lines and line segments. Notice that for a line, the parameter can take on any real value. For a line segment, the parameter takes values in a closed interval $[a, b]$ and the equation should evaluate to one of the endpoints at each endpoint of the interval.

Example 4 is a realatively simple real-world application of parametrized lines. The more interesting versions of this (when the motion is not in a straight line) will show up in chapter 13.

Example 5 is just a computation using the formula for the distance between a point and a line. We will discuss techniques for finding a distance without using this formula, for those of you who (like me) hate memorizing formulas. That said, you may want to drop this formula on your reference sheet.

Examples 6 and 7 find the equations of planes. These are very important when we linearize multivariable functions in chapter 14, so please make sure you understand these.

Examples 8, 9, and 10 involve finding intersections between geometric objects in $\mathbb{R}^3$. The general ideas are important, but there are several different ways to handle the details. Anything that works for you is fine.

Example 11 is a calculation of the distance between a point and a plane. Again, there are several ways to do this. The most important part of example 12 is the (obvious?) fact that the angle between two planes must be congruent to the angle formed by their normal vectors.

Section 12.6 Cylinders and Quadric Surfaces

This section explores many of the common surfaces we will see throughout the course. Cylinders are really just the three-dimensional translations of circles, and you have certainly seen them in high school math classes. The quadric surfaces are essentially the 3D versions of the conic sections which you should have studied in precalculus (circles, parabolas, ellipses, and hyperbolas). The equations of these surfaces all have equations which are polynomial of at most degree two in each variable. For our purposes, it is much more important that you have an idea of what they look like than that you can name them perfectly.

Read through the section, which is not very long. See if you can figure out why the various surfaces look the way they do based on their equations. The more of these you think about, the easier they will be for you.