Calculus 251:C3 Reading Guide - 7/15/2021

Section 16.6 Surface Integrals

In this section, we explore the both types of surface integral, the scalar surface integral and the vector surface integral. The surface area computations were a specific case of scalar surface integrals, the case where the integrand is 1. Note that for scalar surface integrals, the orientation of the surface does not matter. This is analagous to the direction of the parametrization being irrelevant to our computation of scalar line integrals.

We generally think about vector surface integrals as the flow of something from one side of our surface to the other, and these are often referred to as flux integrals. We could be taking about water flowing through a membrane, or electric flux, or the flow of heat through some surface. One important point is that because we are talking about flow through a surface, we are interested in the normal component of the vector field. The tangential component is moving along the surface at the point of tangency, so does not get counted as we are computing flow across the surface. Another point is that we need our surface to be oriented, because we need to know which direction through the surface will be counted as positive flux and which direction will be counted as negative flux.

Find the following definitions/concepts/formulas/theorems:

- surface integral - definition
- surface integral of a scalar function - formulas
- orientation (for a parametrized surface)
- normal component (of a vector field w.r.t. a surface)
- vector surface integral (a/k/a flux integral)
- flux (of a field through a surface)
- formula for flux across a parametrized surface (hidden away in the margin)

Examples 1, 2, 3, and 4 are all surface integrals of scalar functions. Examples 1 and 3 both use parametrizations already found in examples in the previous section, so you may have to flip back a few pages. Yes, this is why scalar surface integrals could reasonable have been covered in the previous section. Example 2 includes an important time-saving observation, and you should look for similar opportunities when they present themselves. Example 4 uses the same special form of the Jacobian for $z = f(x, y)$ that we saw in the previous section.

The section about orientation of a surface is a must-read. Among other things, it discusses the Möbius strip and why it is a non-orientable surface. The basic idea is that you can't talk about the flow from one side to the other, because the surface only has one side. Cool stuff.

Example 5 is a typical example of a flux integral calculation. There are a lot of steps, but they are generally the same steps. Parametrize the surface, find the tangent vectors, find the normal, dot the normal with the field, integrate that (now scalar) function over the parameter domain. Please note that in this example, they compute the unit normal and later notice that $d\sigma$ is equal to the length of the normal times $dx\ dz$. The formula in the margin (or just plain common sense) tells you that you can just use $\vec{F} \cdot (\vec{r}_u \times \vec{r}_v)$ as the integrand to avoid dividing and multiplying by the same quantity. Please do it that way.

Example 6 just takes advantage of the fact that we know that gradients are normal to level surfaces. Otherwise, it is just a normal flux integral calculation.

You can skip the part about Moments and Masses of Thin Shells and examples 7 and 8. You may need them for the homework, but you won't need this section to understand the lecture.