Calculus 251:C3 Reading Guide - 7/13/2021

Section 16.5 Surfaces and Area

When we computed line integrals, we needed parametrized curves along which to integrate. This section is a natural extension of that concept. We will now be parametrizing surfaces and considering integrals over those surfaces. For a curve, we only needed one parameter because a curve is essentially one dimensional (i.e. you can only move forwards or backwards along it) even though the curve itself may have been in 2 or 3 dimensional space. We really treated a curve as a deformed line segment, and worked out how to integrate along it using single-variable calculus with respect to the parameter.

We can think about a surface as a deformed region of a plane, parametrize it using two parameters (because it is essentially a two dimensional object even though it resides in 3 or more dimensions), and use a double integral with respect to those two parameters. In this section, we will only be computing integrals to find surface areas. In the next section, we will consider the integrals of scalar functions over surfaces (the analogue of scalar line integrals) and flux integrals (the analogue of vector line integrals).

Find the following definitions/concepts/formulas/theorems:

- parametrized surface
- parameter domain
- Parametrization formulas for cylinders, spheres
- What would be a good parametrization for the graph of $z = f(x, y)$?
- grid curves
- smoth parametrized surface (same general idea as regular parametrizations for curves, but different details)
- normal vector (same old idea - new context)
- surface area formula (what role is the magnitude of the cross product playing here?)
- surface area differential $d\sigma$
- formula for the surface area of an implicit graph
- formula for the surface area of the graph of $z = f(x, y)$

Examples 1, 2, and 3 are parametrizations of various surfaces. You really need to make sure you understand these, because nothing else in this section (or the remainder of the course) will make sense if you don't.

Read through the bit about normal vectors, tangent planes, and where the surface area formula comes from. These are the pieces we need to put together in order to compute surface area in this section and surface integrals in the next. This discussion should remind you of §15.8 Substitutions in Multiple Integrals. The idea is very similar, in that we are looking at mapping from a plane to a surface, and trying to figure out how the area of a small rectangle scales across the mapping.

Examples 4, 5, and 6 illustrate how to compute surface areas of parametrized surfaces. These are the meat of the section, and you should make every effort to work through them.

The part about surface areas of implicit surfaces (including example 7) is interesting, but not essential to master. You can usually either arrange your parametrization well enough to avoid needing this technique, at least locally.

Please do read through example 8. This is essentially a proof of the formula for the surface area of the graph of $z = f(x, y)$. You will definitely need this formula, and knowing where it comes from will help you understand how to apply it.