Calculus 251:C3 Reading Guide - 7/8/2021

Section 16.3 Path Independence, Conservative Fields, and Potential Functions

This section explores conservative vector fields in some depth. It's not that other vector fields aren't interesting, it's just that many of the vector fields we see in physics are conservative. Gravitational fields and electrostatic fields in particular are conservative, which is one of the reasons why we study conservative vector fields in such detail. As you will see, conservative vector fields have some very nice properties which will make much of our analysis easier.

Notational reminder: $\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ and $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ really mean the same thing. The circle on the integral sign is just a way to indicate that we are integrating on a *closed* curve.

Find the following definitions/concepts/formulas/theorems:

- gravitational field/electric field
- path independence
- conservative (for a vector field)
- potential function
- piecewise smooth (we have seen this before)
- connected/simply connected (this time for a domain in $\mathbb{R}^2$ or $\mathbb{R}^3$)
- Theorem: Fundamental Theorem of Line Integrals (a/k/a Fund. Thm. for Conservative Vector Fields)
- Theorem: Conservative Fields are Gradient Fields
- Theorem: Loop Property of Conservative Fields
- Not in book, but useful: equipotential curves are the level curves of a potential function
- technique for finding potential functions (we discussed this in class when we studied gradients)
- component test for conservative fields (IMPORTANT: note that the field must be defined on an open *simply connected* domain)
- differential form/exact differential form (not terribly important for this class, but you will see it when you take 244/252 at Rutgers or the differential equations class at your home university)

I don't say this often, but you should definitely read the proofs of the three theorems. The first and third proofs are easy, the second one is harder but still comprehensible, and they link the new results in this chapter to previous results that we have used quite a bit.

Examples 1 and 2 are calculations that are *much* simpler than the line integrals from the previous section. The reason they are simpler is because we know the vector fields are conservative. Any time you are trying to compute a vector line integral, you should ask yourself, “Is my vector field conservative?” If the answer is yes, you will end up doing a lot less work to get the same result. You should definitely make sure you understand these.

Examples 3, 4, and 5 are typical explorations of whether a given vector field is conservative. Example 3 looks like what we did in class when we discussed gradients. Example 4 shows how to apply the component test to show that a field is not conservative. Example 5 is more work, but it is very important. Note that the component test is not conclusive when the field’s natural domain is not simply connected!

Example 6 is the simplest possible exact differential forms example. If it were any more complicated, I would tell you to skip it.