Calculus 251:C3   Reading Guide - 7/6/2021

Section 16.1 Line Integrals of Scalar Functions

What is a scalar line integral really? We are integrating a multivariable function $f(\vec{x})$ along some (parametrized) curve in $\mathbb{R}^n$. If that curve *happens* to be the x-axis, we are just doing single-variable calculus. When we thought about the chain rule for paths, we were trying to think "along the curve" and talk about the change in function value from the perspective of walking along the curve. This is very much the same idea. Notice that the magnitude of $\vec{v} = \vec{x}\,'$ is accounting for the rate at which we are moving along the curve. This is hopefully reminiscent of arc length parametrization.

Find the following definitions/concepts/formulas/theorems:

- line integral
- subarc (a/k/a arc length differential OR line element)
- definition of scalar line integral
- evaluating a Scalar Line Integral
- additivity (used for piecewise smooth curves)
- path dependence
- Mass and Moment calculations (OPTIONAL)
- interpretation of line integrals when the curve lies in the xy-plane

The discussion of scalar line integrals leading up to the "How to Evaluate a Line Integral" box is really an adaptation of Riemann sums to the new context. It's an interesting read and may help develop your intuition for how and why line integrals work. Not absolutely essential, but probably a good idea to spend a little time on it.

Examples 1, 2, and 3 are basic examples of scalar line integrals. You should definitely make sure you understand these.

The section on mass and moment calculations is optional, but very useful for physicists and engineers. You should read this section after going back and reading the first few pages of §15.6 if you are interested. The ideas for center of mass, and first/second moments are essentially the same whether we are talking about planar objects (double integrals), solid objects (triple integrals), or obects like flexible wires which we can represent with parametrized curves (scalar line integrals).

The most important part of the subsection on line integrals in the plane is that it may give you an easy way to visualize what is happening if you are a spatial thinker (rather than a symbolic thinker). This visual is accumulation of area, so it ties back to what you saw in calc 1. You can think about the integral of a density function (even if the curve is not in the plane) as an accumulation of mass, so the mass density is the "height" and you just roll the curve (wire?) out along the x-axis to get the same visual.