Calculus 251:C3 Reading Guide - 7/1/2021

Section 15.8 Substitutions in Multiple Integrals

This section discusses how to change variables in a multiple integral. Among other things, this section shows us *why* the integral formulas in §15.4 and §15.7 contained the extra factor of r (or $\rho^2 \sin(\phi)$ for spherical). The motivation for being able to change coordinates generally is the same as the motivation for polar, cylindrical, and spherical coordinate systems: we hope that the change of coordinates will make the limits of integration, the integrand, or both simpler to handle. We are essentially trading up-front setup costs (finding the mapping and computing the Jacobian) for a smoother integration.

Find the following definitions/concepts/formulas/theorems:

- coordinate transformation (a/k/a map or mapping)
- one-to-one transformation/map (means the same thing it did in high school algebra)
- image/preimage
- Jacobian (a/k/a Jacobian determinant - you should notice that there are several notations)
- Theorem: Substitution for Double Integrals (a/k/a Change of Variables Formula)
- polar coordinates map (in Example 1)
- linear transformation/map
- change of variables formula for three variables
- Jacobian calculations for cylindrical and spherical coordinates

Example 1 just computes the Jacobian for polar coordinates. We showed why the r was there geometrically (and informally), and this is a completely independent justification for the presence of r in $r\,dr\,d\theta$.

Examples 2 and 3 work with linears map between coordinate systems. These are fundmental examples, and you should make sure you understand them. We will do more examples of linear maps in class. What do you notice about the Jacobians? Do you think that will be true for any linear map?

Example 4 is a more complicated example, mostly because the map is nonlinear. Make sure that you understand the process, even if the details are a little hazy.

Example 5 is a linear transformation, but with the twist that there are three independent variables. The motivation is that we are mapping a slanted solid region into a rectangular one because the integration process is simpler.