Calculus 251:C3 Reading Guide - 6/30/2021

Section 15.7 Triple Integrals in Cylindrical and Spherical Coordinates

We have already seen double integrals where the domain is expressed in polar coordinates. This section introduces triple integrals in both cylindrical and spherical coordinates. Again, the motivation is that many situations have a natural symmetry to them which make non-rectangular coordinate systems more convenient. We may get a simpler integrand, less complicated limits of integration, or both.

One very important thing to notice is that there are extra factors which show up in the integrand when you change coordinate systems. So not only do you need to rewrite the function using techniques similar to the ones we discussed for polar integrals, you also get an extra factor of r (for cylindrical, just like it was for polar) or $\rho^2 \sin\phi$ (for spherical). We will discuss the reasons for that in the lecture after this one (§15.8 Substitution in Multiple Integrals). If you can't wait, example 1 in §15.8 discusses the r for polar coordinates, and the computations for cylindrical and spherical are on p. 959 in the subsection Substitutions in Triple Integrals. For now, make sure you notice that those extra factors are there and that you include them.

Find the following definitions/concepts/formulas/theorems:

- In lecture, we will discuss the terms "radially simple," "centrally simple," and "axial symmetry." Think about what these terms should mean, and keep them in the back of your mind as you read the section.
- definition of Cylindrical Coordinates
- equations relating rectangular and cylindrical coordinates
- cylindrical wedge
- definition of triple integral in cylindrical coordinates
- definition of Spherical Coordinates
- equations relating rectangular, cylindrical, and spherical coordinates
- spherical wedge
- definition of triple integral in spherical coordinates

I don't really have a ton to say to guide you through this section. Each of the two integrals (cylindrical and spherical) has two subsections, one on the theory and one "How-To" guide. The former subsections give you definitions behind each type of integral (including construction of the Riemann sums) and *when you should use it*. The examples and the diagrams are all useful, and you should force yourself to work through as many of them as you can.

As with many of the multiple integrals we have already seen, the hard part is setting up the integral and figuring out which of the six possible orders of integration will be cleanest.

Honestly, though, the setup will usually be done in the same order every time because thinking about where the limits of integration should be is easiest with z or ρ as the innermost integral. Much of the rest is just slogging through computations, so it is tedious but still important to get right.