Calculus 251:C3 Reading Guide - 6/28/2021

Section 15.5 Triple Integrals in Rectangular Coordinates

Here, we have a section which is (appropriately) mostly examples. We are going to look at integrals of the form $\displaystyle\iiint\limits_{\mathcal{D}} f(x, y, z)\ dV$ where $\mathcal{D}$ is some region in $\mathbb{R}^3$. There is not too much difference between double and triple integrals, but you do have more choices and additional calculations.

Just to provide some motivation, triple integrals have many useful applications. In this section, we will see that we can use a triple integral to find the average value of f on $\mathcal{D}$. We will be skipping §15.6 on applications of multiple integrals, but I strongly suggest looking at it when you have a chance. In that section, we see that we can use triple integrals to calculate mass given a density function, find the center of mass of an object, and find moments of inertia. Triple integrals are also used in calculations of total charge, heat content, and total energy. Multiple integrals are also critical for many probability applications.

Find the following definitions/concepts/formulas/theorems:

- integrable (same idea as definition from §15.1)
- volume as a triple integral
- The book mentions Fubini's Theorem for Triple Integrals, but doesn't state it. Any idea what it would look like?
- number of orders in which you can write a triple integral
- Think about our class discussion of "horizontally simple" and "vertically simple" regions for double integrals. Can you find analagous definitions for x-simple, y-simple, z-simple for a region in $\mathbb{R}^3$?
- projection (onto a coordinate plane) - the book calls this a "shadow"
- average value of a function in space

There is a lot of discussion before the first example. Much of it is just extending the concepts of §15.1 into three dimensions. All of it is worth reading, but pay particular attention to the part titled "Finding Limits of Integration in the Order $dz\ dy\ dx$".

Examples 1 and 2 are detailed examples of how to set up the limits of integration. If I am to be completely honest with you, being able to set the integral up properly is *more* important than being able to evaluate it correctly. If you can do the setup, a computing package can give you a number. If you can't do the setup, a computing package will not help you. That said, yes you will have to calculate triple integrals by hand on quizzes and exams. Sorry. :(

Example 3 finally computes a triple integral using the setup from the previous example.

Then they work through how you would change to one of the other possible orders of integration. They evaluate the integral again and get the same answer, thankfully.

Example 4 is a much more challenging setup. Try to work through it, but don't panic if you don't fully understand it. This example is one of the reasons that we have other coordinate systems and the general change of variables formula that we will see in the last two sections of this chapter.

Example 5 is an average temperature calculation, and a much simpler integral than several of the previous examples. This one you should definitely make sure you understand.