Calculus 251:C3 Reading Guide - 6/22/2020

Section 15.3 Area by Double Integration

The general idea of this section is that if you evaluate $\iint_{\mathcal{R}} 1\, dA$, you get the area of $\mathcal{R}$. This is the equivalent of saying that if you integrate 1 over a closed interval on the real line, you get back the width of the interval. The other main idea is that if you integrate a function of two variables over a region and divide by the area of the region, you get the average value of the function. This is completely analagous to the average value of a function from single-variable calculus.

Find the following definitions/concepts/formulas/theorems:

- area formula
- average value (of a function of two variables)

Examples 1, 2, and 3 are area calculations. You should try to work through all of them. There are some figures to guide you, and none of them are terrible. Notice that in example 2, if you use horizontal slices you will need two integrals (because the function definining the left boundary changes). If you use vertical slices, you only need one integral because the top and bottom boundaries are each a single function. This happens frequently. There will even be times when you can't get a single integral either horizontally or vertically, so we make up a new coordinate system just so we can integrate over a rectangle! (This is pretty much the entire point of §15.8)

Example 4 is an average value calculation. Why did they choose to integrate in the order they did? What would happen if you tried this integral in the other order?

Section 15.4 Double Integrals in Polar Form

This section discusses double integrals where the domain is expressed in polar coordinates. The motivation is that many situations have a natural symmetry to them which make non-rectangular coordinate systems more convenient. We may get a simpler integrand, less complicated limits of integration, or both.

One very important thing to notice is that there are extra factors which show up in the integrand when you change coordinate systems. So not only do you need to rewrite the function using the techniques we have already discussed, you also get an extra r in the integrand for polar integrals. As you will see later, we get an r in cylindrical coordinates or $\rho^2 \sin\phi$ in spherical coordinates. We will discuss the reasons for that when we cover §15.8 (Substitutions in Multiple Integrals). If you can't wait, example 1 in §15.8 discusses the r for polar coordinates. For now, make sure you notice that those extra factors are there and that you include them.

Find the following definitions/concepts/formulas/theorems:

- polar rectangle

- Finding limits of integration for a polar function
- Area in polar coordinates
- Changing from Cartesian (rectangular) coordinates to polar

I don't really have a ton to say to guide you through this section. All of the examples are useful, and you should work through as many of them as you can. I have faith in you.