Calculus 251:C3 Reading Guide - 6/21/2021

Section 14.8 Lagrange Multipliers

The basic idea of Lagrange multipliers is this: if you want to maximize a function, you *want* to move in the direction of the gradient from wherever you are. If you are subject to a constraint, you may not be able to move in the direction of the gradient while staying "in bounds" for the constraint. So you do the best you can and move along your constraint curve (or surface) in the direction closest to the gradient direction. When you get to a place where the gradient is orthogonal to the constraint where you are, moving in either direction won't increase the function value (recall: orthogonal to gradient=along level curve/surface). So where you are is at least a local maximum subject to the constraint.

Find the following definitions/concepts/formulas/theorems:

- constraint (you have seen this before)
- Lagrange multiplier - general concept
- Orthogonal Gradient Theorem
- Lagrange conditions (in the book this is called The Method of Lagrange Multipliers)
- critical point/critical value (specifically for optimization with a constraint)
- Lagrange multipliers with two constraints

The first two examples are very long, because they are trying to solve a constrained optimization problem *without* using Lagrange multipliers. Try to follow along all the way through the examples, but understand that the main purpose of these examples is to motiviate the technique about to be introduced. Lagrange multipliers take care of many of the considerations in these examples without having to think deeply into the geometry on every problem.

The proof of the Orthogonal Gradient Theorem is just an application of the chain rule for paths. You can skip it if you want to.

Examples 3 and 4 are both long, but they are about as simple as Lagrange Multiplier examples can be. The setup isn't too bad in either example, and they perform the same steps in the same order. You should certainly try to work your way through them. One important note is that we often end up having to solve systems of nonlinear equations as one step of a Lagrange problem. Unlike linear systems, there is no easy algorithm that will work on every system - sometimes you just have to play around with the equations until something good happens. It is very common that we need to break into cases (as in Example 3). You should also realize that the number of equations will always be one more than the number of independent variables for the function, because the Lagrange multiplier (λ) is also an unknown. For a two-variable function, you will have three equations (in x, y, λ).

Example 5 gets tricky because we have more than one constraint. When we have multiple constraints, we have to set up one Lagrange multiplier for each constraint. In this case, the Lagrange condition is that the gradient of our objective function is a linear combination of the gradients of the constraints. You may not want to look at this one until after the lecture. Hopefully it will make some sense then.