

## Section 14.7 Extreme Values and Saddle Points

This section is essentially the multivariable version of finding relative extrema using derivatives. As you are reading through it, you should always be thinking, “What did this look like in Calculus I? What is different now?”. It is possible to extend these techniques to functions of three or more variables, but that is beyond the scope of this course. If you are really curious (and I mean *really* curious), you could look at the Applications section of [this Wikipedia page](#) for a basic introduction of what happens with more variables. Click at your own risk.

Find the following definitions/concepts/formulas/theorems:

- local minimum/maximum
- local (a/k/a relative) extrema
- Theorem: First Derivative Test
- critical point
- saddle point
- Theorem: Second Derivative Test (for functions of exactly two variables)
- discriminant (a/k/a Hessian determinant)
- absolute extrema on closed, bounded domains
- Review terms from previous sections: bounded, interior point, boundary point

The proof of the First Derivative Test is really just applying the one-variable version to both partials. Not too bad. It’s more important for you to know the theorem than its proof.

The proof of the second derivative test is in section 14.9, which we will not cover in this class. If you plan to be a math major (or are really curious), you should spend some time considering it. If neither of those applies to you, you can skip it.

Examples 1 and 2 are very basic problems for this topic. Each of these examples has a single critical point. One of them is a minimum, the other is a saddle point.

Examples 3, 4, and 5 are typical problems, and would all be fair game for an exam. You are given a function. You compute partials, find critical points, compute second-order partials, find the discriminant, use it to classify the critical points.

Example 6 isn’t terrible because the domain is a triangle with two sides parallel to the coordinate axes, which makes the calculations on the boundary easy. You should definitely make sure you can do a question similar to this one. Example 7 is annoying and cool at the same time, since you have to figure out what the domain is based on the physical situation.