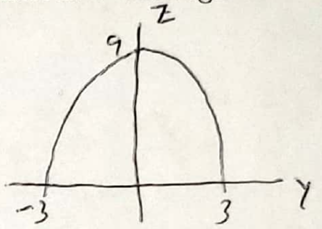
Name: Key

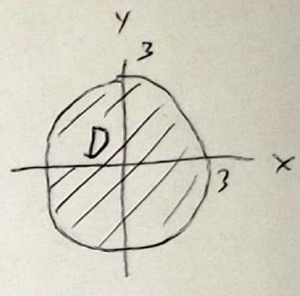
Calculus 251:C3 Quiz #26 - 7/21/2021 Topic: Section 16.8

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts (1) Let $\vec{F} = (e^{yz} - 4xz)\,\hat{\imath} + (e^{xz} + 4yz)\,\hat{\jmath} + (x^2 + y^2)\,\hat{\mathbf{k}}$, and let $\mathcal{S}$ be the part of the paraboloid $z = 9 - x^2 - y^2$ above the xy-plane, oriented with upward-pointing normal.

Use the Divergence Theorem to calculate the flux of $\vec{F}$ through $\mathcal{S}$.





∂S is the circle in the xy-plane of radius 3 centered at the origin.

Let D be the disk in the xy-plane enclosed by ∂S. Let W be the region enclosed by S+D.

$\operatorname{div}(\vec{F}) = (-4z) + (4z) + 0 = 0$, so $\iiint_W \operatorname{div}(\vec{F})\,dV = \iiint_W 0\,dV = 0$

Now we orient D with an upward-pointing normal, so $\partial W = S - D$
(also inward)

so $0 = \iiint_W \operatorname{div}(\vec{F})\,dV = \oiint_{\partial W} \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot d\vec{S} - \iint_D \vec{F}\cdot d\vec{S} \Rightarrow \iint_S \vec{F}\cdot d\vec{S} = \iint_D \vec{F}\cdot d\vec{S}$

Parametrize D: $G(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle$, $0 \le r \le 3$, $0 \le \theta \le 2\pi$

$\vec{T}_r = \langle \cos\theta, \sin\theta, 0\rangle$, $\vec{T}_\theta = \langle -r\sin\theta, r\cos\theta, 0\rangle$ $\vec{N} = \vec{T}_r \times \vec{T}_\theta = \langle 0, 0, r\rangle$
upward ✓

$\vec{F}(G(r,\theta)) = \langle \text{blob}, \text{blob}, r^2\rangle$

$\vec{F}\cdot\vec{N} = \langle \text{blob}, \text{blob}, r^2\rangle \cdot \langle 0,0,r\rangle = r^3$

$$\iint_D \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^3 r^3\,dr\,d\theta = 2\pi\left(\frac{r^4}{4}\right)\Big|_{r=0}^{r=3} = 2\pi\left(\frac{81}{4}\right) = \frac{81\pi}{2}$$

$$\Rightarrow \iint_S \vec{F}\cdot d\vec{S} = \frac{81\pi}{2}$$