Name: Key

Calculus 251:C3 **Quiz #23 - 7/15/2021** **Topic: Section 16.5**

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts

(1) A thin sheet is in the shape of the part of the surface $z = 9 - x^2 - y^2$ above the xy-plane and below the plane $z = 5$.

(a) Parametrize the surface (choose wisely).

(b) Compute the tangent and normal vectors.

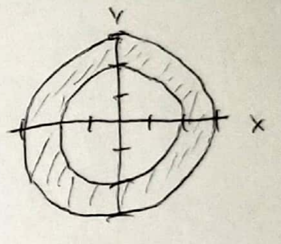
(c) Calculate the surface area of the sheet.

(d) **Bonus:** If the mass density of the sheet is given by $\delta(x, y, z) = \dfrac{5}{\sqrt{4x^2 + 4y^2 + 1}}$, find the mass of the sheet.

$5 = 9 - x^2 - y^2$

$x^2 + y^2 = 4, \ z = 5$

intersection curve



It seems natural to use polar/cylindrical

a) $G(r, \theta) = (r\cos\theta, r\sin\theta, 9 - r^2)$ $\quad 2 \le r \le 3$, $\ 0 \le \theta \le 2\pi$

b) $\vec{T}_r = \langle \cos\theta, \sin\theta, -2r \rangle$

$\vec{T}_\theta = \langle -r\sin\theta, r\cos\theta, 0 \rangle$

$\vec{N}(r,\theta) = \langle 2r^2\cos\theta, 2r^2\sin\theta, r \rangle$ $\qquad \|\vec{N}\| = \sqrt{4r^4 + r^2} = r\sqrt{4r^2+1}$

c) $$\text{Area}(S) = \int_0^{2\pi}\int_2^3 r\sqrt{4r^2+1}\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{1}{8}\cdot\frac{2}{3}(4r^2+1)^{3/2}\Big|_{r=2}^{r=3} d\theta = \int_0^{2\pi} \frac{1}{12}\left(37^{3/2} - 17^{3/2}\right) d\theta$$

$$= 2\pi \cdot \frac{1}{12}\left(37^{3/2} - 17^{3/2}\right) = \frac{\pi}{6}\left(37^{3/2} - 17^{3/2}\right)$$

d) $$\iint_S \delta\, dS = \int_0^{2\pi}\int_2^3 \left(\frac{5}{\sqrt{4r^2+1}}\right)\left(r\sqrt{4r^2+1}\right) dr\, d\theta$$

$$= \int_0^{2\pi}\int_2^3 5r\, dr\, d\theta = 2\pi \cdot \frac{5r^2}{2}\Big|_{r=2}^{r=3} = 5\pi(9-4) = 25\pi$$