Name: Key

Calculus 251:C3 **Quiz #22 - 7/13/2021** **Topic: Section 16.4**

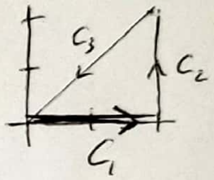
Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts (1) Let $\vec{F} = \langle e^x - y, y + \sin x \rangle$, and let $\mathcal{D}$ be the region enclosed by the triangle with vertices $(0,0), (2,0)$, and $(2,2)$. Verify the Circulation-Curl Form of Green's Theorem, i.e. show that $\oint_{\partial \mathcal{D}} \vec{F} \cdot d\vec{r} = \iint_{\mathcal{D}} \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA.$

Hint: The right hand side is an easier setup. The left hand side is easier to integrate.

LHS

Direct Computation



C_1: $\vec{r}_1(t) = \langle t, 0 \rangle,\ t \in [0,2]$ $\quad \vec{F}(\vec{r}_1(t)) = \langle e^t, \sin t \rangle$

$\vec{r}_1{}'(t) = \langle 1, 0 \rangle$ $\quad \vec{F}(\vec{r}_1(t)) \cdot \vec{r}_1{}'(t) = e^t$

C_2: $\vec{r}_2(t) = \langle 2, t \rangle,\ t \in [0,2]$ $\quad \vec{F}(\vec{r}_2(t)) = \langle e^2 - t, t + \sin 2 \rangle$

$\vec{r}_2{}'(t) = \langle 0, 1 \rangle$ $\quad \vec{F}(\vec{r}_2(t)) \cdot \vec{r}_2{}'(t) = t + \sin 2$

C_3: $\vec{r}_3(t) = \langle 2-t, 2-t \rangle$ $\quad \vec{F}(\vec{r}_3(t)) = \langle e^{2-t} - 2 + t, 2 - t + \sin(2-t) \rangle$

$\vec{r}_3{}'(t) = \langle -1, -1 \rangle$ $\quad \vec{F}(\vec{r}_3(t)) \cdot \vec{r}_3{}'(t) = -e^{2-t} + 2 - t - 2 + t - \sin(2-t) = -e^{2-t} - \sin(2-t)$

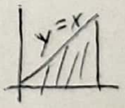
Note: each $t \in [0,2]$, so I can combine into one integral

$$\oint_{\partial D} \vec{F} \cdot d\vec{r} = \int_0^2 \left[e^t + t + \sin 2 - e^{2-t} - \sin(2-t) \right] dt = \left[e^t + \frac{t^2}{2} + t \sin 2 + e^{2-t} - \cos(2-t) \right]_0^2$$

$$= (e^2 + 2 + 2\sin 2 + 1 - 1) - (1 + 0 + 0 + e^2 - \cos 2) = \boxed{2\sin 2 + \cos 2 + 1}$$

RHS

$$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = \cos x + 1$$



$0 \le x \le 2$

$0 \le y \le x$

$$\iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA = \int_0^2 \int_0^x (\cos x + 1)\, dy\, dx = \int_0^2 \left[y \cos x + y \right]_{y=0}^{y=x} dx$$

$$= \int_0^2 (x \cos x + x)\, dx$$

$\int x \cos x\, dx = x \sin x + \cos x + C$

$u = x \quad v = \sin x$

$du = dx \quad dv = \cos x\, dx$

$$= \left[x \sin x + \cos x + \frac{x^2}{2} \right]_{x=0}^{x=2} = (2\sin 2 + \cos 2 + 2) - (0 + 1 + 0)$$

$$= \boxed{2\sin 2 + \cos 2 + 1}$$

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