Name: Key

Calculus 251:C3 Quiz #21 - 7/12/2021 Topic: Section 16.3

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts (1) Let C be the part of an elliptical helix parametrized by

$$\vec{r}(t) = \left\langle 3\cos t, 2\sin t, \frac{2}{3\pi}t \right\rangle, \ 0 \le t \le \frac{9\pi}{2}.$$

Let $\vec{F} = \langle 2xy^3 - 3z^2, 3x^2y^2 + 2yz, y^2 - 6xz \rangle$ be a force field which acts on a particle traveling along the path C. Calculate the work done on the particle by $\vec{F}$.

Note: There are easy ways to do this problem and there are hard ways. For your sanity and for mine, please don't do it the hard way.

$$\text{curl}(\vec{F}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy^3 - 3z^2 & 3x^2y^3 + 2yz & y^2 - 6xz \end{vmatrix} = (2y - 2y)\hat{i} - (-6z - (-6z))\hat{j} + (6xy^2 - 6xy^2)\hat{k} = \vec{0}$$

$\vec{F}$ is defined on $\mathbb{R}^3$, which is simply connected, so $\vec{F}$ is conservative.

$f = x^2y^3 - 3xz^2 + y^2z$ is a potential for $\vec{F}$

$\vec{r}(0) = \langle 3, 0, 0 \rangle$ and $\vec{r}\left(\frac{9\pi}{2}\right) = \langle 0, 2, 3 \rangle$

$$\int_C \vec{F} \cdot d\vec{r} = f\left(\vec{r}\left(\tfrac{9\pi}{2}\right)\right) - f(\vec{r}(0)) \quad \text{because } \vec{F} \text{ is cons.}$$

$$= f(0, 2, 3) - f(3, 0, 0)$$

$$= (0 - 0 + 12) - (0 - 0 + 0) = \boxed{12}$$