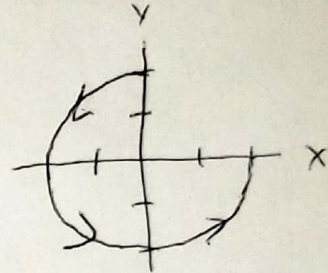**Calculus 251:C3   Quiz #20 - 7/8/2020   Topic: Section 16.2**

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. <u>Show and label all of your work.</u> Responses with no work may receive no credit even if the answer is correct.

10 pts

(1) Let C be the three quarters of the circle in the xy-plane with radius 2, centered at the origin, starting at $(0, 2, 0)$ and ending at $(2, 0, 0)$. Let $\vec{F} = \langle xy, x^2, e^z \rangle$ be a force field which acts on a particle traveling along the path C.

Calculate the work done on the particle by $\vec{F}$.

$$\vec{F}(\vec{r}(t)) = \langle 4\sin t \cos t, 4\cos^2 t, 1 \rangle$$

$$\vec{r}(t) = \langle 2\cos t, 2\sin t, 0 \rangle$$
$$t \in \left[\frac{\pi}{2}, 2\pi\right]$$
$$\vec{r}'(t) = \langle -2\sin t, 2\cos t, 0 \rangle$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_{\pi/2}^{2\pi} \langle 4\sin t \cos t, 4\cos^2 t, 1 \rangle \cdot \langle -2\sin t, 2\cos t, 0 \rangle \, dt$$

$$= \int_{\pi/2}^{2\pi} -8\sin^2 t \cos t + 8\cos^3 t \, dt$$

$$= \int_{\pi/2}^{2\pi} -8\sin^2 t \cos t + 8\cos t - 8\sin^2 t \cos t \, dt$$

Using
$$\cos^3 t = \cos t (1 - \sin^2 t)$$

$$= \int_{\pi/2}^{2\pi} 8\cos t - 16\sin^2 t \cos t \, dt$$

$$= 8\sin t - \frac{16}{3}\sin^3 t \Big|_{\pi/2}^{2\pi}$$

$$= (0 - 0) - \left(8 - \frac{16}{3}\right) = -\left(\frac{24}{3} - \frac{16}{3}\right) = \boxed{-\frac{8}{3}}$$