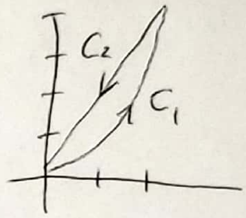
Name: Key

Calculus 251:C3 **Quiz #19 - 7/7/2021** **Topic: Section 16.1**

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts (1) Calcuate $\int_C (x + \sqrt{y})\, ds$ where C is the path starting at the origin, moving along the curve $y = x^2$ to the point $(2, 4)$, then returning to the origin along a straight line segment.

$C_1 : \vec{r}_1(t) = \langle t, t^2 \rangle,\ t \in [0,2]$



$C_2 : \vec{r}_2(t) = \langle 2-t, 4-2t \rangle,\ t \in [0,2]$

$|\vec{r}_1\,'| = |\langle 1, 2t \rangle| = \sqrt{1+4t^2}$ $\quad f(\vec{r}_1(t)) = t + \sqrt{t^2} = 2t$

$|\vec{r}_2\,'| = |\langle -1, -2 \rangle| = \sqrt{5}$ $\quad f(\vec{r}_2(t)) = 2 - t + \sqrt{4-2t}$

$$\int_{C_1} (x+\sqrt{y})\, ds = \int_0^2 (2t)\sqrt{1+4t^2}\, dt = \frac{1}{4}\cdot\frac{2}{3}(1+4t^2)^{3/2}\Big|_0^2 = \frac{1}{6}\left(17^{3/2} - 1\right)$$

$$\int_{C_2} (x+\sqrt{y})\, ds = \int_0^2 \left(2-t+\sqrt{4-2t}\right)(\sqrt{5})\, dt = \sqrt{5}\left(2t - \frac{t^2}{2} - \frac{1}{2}\cdot\frac{2}{3}(4-2t)^{3/2}\right)\Big|_0^2$$

$$= \sqrt{5}\left[(4-2-0) - \left(0-0-\frac{1}{3}(4)^{3/2}\right)\right] = \sqrt{5}\left(2+\frac{8}{3}\right) = \frac{14\sqrt{5}}{3}$$

so $$\int_C (x+\sqrt{y})\, ds = \frac{17\sqrt{17}}{6} - \frac{1}{6} + \frac{14\sqrt{5}}{3}$$

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