Name: Key

Calculus 251:C3 Quiz #17 - 6/24/2020 Topic: Section 15.4

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts (1) Let $\mathcal{W}$ be the region bounded above by the sphere $x^2+y^2+z^2=16$ and below by the plane $z=2$.

(a) Write and evaluate an integral using cylindrical coordinates for the volume of $\mathcal{W}$.

(b) Write and evaluate an integral using spherical coordinates for the volume of $\mathcal{W}$.

(a) sphere $r^2+z^2=16$ intersection at $x^2+y^2+4=16$

$z=\sqrt{16-r^2}$ $\quad x^2+y^2=12$

$r=\sqrt{12}$

$$\int_0^{2\pi}\int_0^{\sqrt{12}}\int_2^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^{\sqrt{12}}\left(r\sqrt{16-r^2}-2r\right)dr\,d\theta = \int_0^{2\pi}\left(\left(-\tfrac{1}{3}(16-r^2)^{3/2}-r^2\right)\Big|_0^{\sqrt{12}}\right)d\theta$$

$$= \int_0^{2\pi}\left[\left(-\tfrac{8}{3}-12\right)-\left(-\tfrac{64}{3}\right)\right]d\theta = \int_0^{2\pi}\tfrac{20}{3}\,d\theta = \tfrac{40}{3}\pi$$

(b) $\rho=4$ (sphere), $\rho\cos\varphi=2 \Rightarrow \rho=2\sec\varphi$ (plane) intersection at $4=2\sec\varphi \Rightarrow \sec\varphi=2 \Rightarrow \varphi=\frac{\pi}{3}$

$$\int_0^{2\pi}\int_0^{\pi/3}\int_{2\sec\varphi}^{4}\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta = \int_0^{2\pi}\int_0^{\pi/3}\sin\varphi\left(\tfrac{\rho^3}{3}\Big|_{2\sec\varphi}^{4}\right)d\varphi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/3}\sin\varphi\left(\tfrac{64}{3}-\tfrac{8}{3}\sec^3\varphi\right)d\varphi\,d\theta = \int_0^{2\pi}\int_0^{\pi/3}\left(\tfrac{64}{3}\sin\varphi-\tfrac{8}{3}\sec^2\varphi\tan\varphi\right)d\varphi\,d\theta$$

$$= \int_0^{2\pi}\left(-\tfrac{64}{3}\cos\varphi-\tfrac{4}{3}\tan^2\varphi\right)\Big|_0^{\pi/3}d\theta = 2\pi\left[\left(-\tfrac{64}{3}\cdot\tfrac{1}{2}-\tfrac{4}{3}(\sqrt{3})^2\right)-\left(-\tfrac{64}{3}-0\right)\right]$$

$$= 2\pi\left(-\tfrac{32}{3}-\tfrac{12}{3}+\tfrac{64}{3}\right) = 2\pi\left(\tfrac{20}{3}\right) = \tfrac{40}{3}\pi$$