

Name: Key

Calculus 251:C3 Quiz #16 - 6/30/2021 Topic: Section 15.5

Instructions. Answer the question in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts

(1) Let W be the region in the first octant bounded by $x + 3y + 2z = 6$ and the three coordinate planes. Find $\iiint_W x^2 dV$

coordinate planes. Find $\iiint_W x^2 dV$

$$\int_0^6 \int_0^{2-x/3} \int_0^{3-\frac{x}{2}-\frac{3y}{2}} x^2 dz dy dx$$

$$= \int_0^6 \int_0^{2-\frac{x}{3}} x^2 z \Big|_{z=0}^{z=3-\frac{x}{2}-\frac{3y}{2}} dy dx$$

$$= \int_0^6 \int_0^{2-\frac{x}{3}} 3x^2 - \frac{1}{2}x^3 - \frac{3}{2}x^2 y dy dx$$

$$= \int_0^6 3x^2 y - \frac{1}{2}x^3 y - \frac{3}{4}x^2 y^2 \Big|_{y=0}^{y=2-\frac{x}{3}} dx$$

$$= \int_0^6 3x^2(2-\frac{x}{3}) - \frac{1}{2}x^3(2-\frac{x}{3}) - \frac{3}{4}x^2(4-\frac{4x}{3}+\frac{x^2}{9}) dx$$

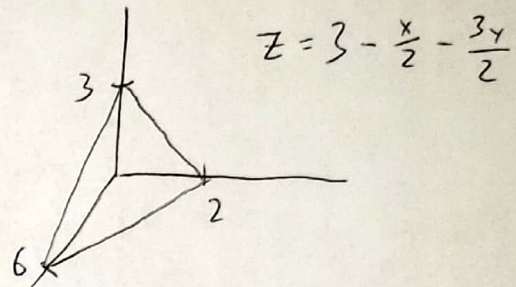
$$= \int_0^6 6x^2 - x^3 - x^3 + \frac{1}{6}x^4 - 3x^2 + x^3 - \frac{1}{12}x^4 dx$$

$$= \int_0^6 3x^2 - x^3 + \frac{1}{12}x^4 dx$$

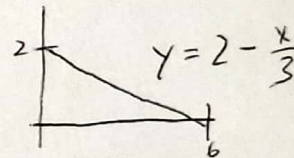
$$= x^3 - \frac{1}{4}x^4 + \frac{1}{60}x^5 \Big|_0^6 = 216 - \frac{1296}{4} + \frac{1296 \cdot 6}{60}$$

$$= 216 - 324 + 129.6$$

$$= 345.6 - 324 = 21.6 = \frac{108}{5}$$



project into xy -plane



$$(2 - \frac{x}{3})^2 = 4 - \frac{4x}{3} + \frac{x^2}{9}$$