

Name: ~~Key~~ Key

Calculus 251:C3 Quiz #15 - 6/28/2021 Topic: Sections 15.3-15.4

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

5 pts

- (1) Find the average value of $f(x, y) = 2xy$ over the region \mathcal{R} where \mathcal{R} is the rectangle $[1, 4] \times [0, 3]$. Square has area 9, so

$$\begin{aligned}\bar{f}(x, y) &= \frac{1}{9} \int_1^4 \int_0^3 2xy \, dy \, dx = \frac{1}{9} \int_1^4 xy^2 \Big|_0^3 \, dx \\ &= \frac{1}{9} \int_1^4 9x \, dx = \int_1^4 x \, dx = \frac{x^2}{2} \Big|_1^4 = \frac{16}{2} - \frac{1}{2} = \frac{15}{2}\end{aligned}$$

5 pts

- (2) Evaluate $\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} e^{x^2+y^2} \, dy \, dx$.

The domain of integration is the circle of radius 3 centered at the origin

$$\begin{aligned}\int_0^{2\pi} \int_0^3 e^{r^2} \cdot r \, dr \, d\theta &= \frac{1}{2} \int_0^{2\pi} \int_0^3 e^{r^2} (2r \, dr) \, d\theta \\ &= \frac{1}{2} \int_0^{2\pi} (e^{r^2}) \Big|_0^3 \, d\theta = \frac{1}{2} \int_0^{2\pi} (e^9 - 1) \, d\theta \\ &= \pi(e^9 - 1)\end{aligned}$$