Calculus 251:C3   Quiz #14 - 6/24/2021   Topic: Sections 15.1-15.2

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

5 pts   (1) Evaluate $\iint_R x + y \, dA$ where R is the rectangle $[1, 2] \times [3, 4]$.

$$\int_1^2 \int_3^4 (x+y) \, dy \, dx = \int_1^2 (xy + \tfrac{1}{2}y^2)\Big|_{y=3}^{y=4} \, dx = \int_1^2 \left[(4x+8)-(3x+\tfrac{9}{2})\right] dx$$

$$= \int_1^2 \left(x + \tfrac{7}{2}\right) dx = \tfrac{1}{2}x^2 + \tfrac{7}{2}x\Big|_1^2 = (2+7) - \left(\tfrac{1}{2}+\tfrac{7}{2}\right) = 9 - 4 = 5$$

5 pts   (2) Evaluate $\int_0^2 \int_0^{4-x^2} \dfrac{xe^{2y}}{4-y} \, dydx$ by switching the order of integration.

$$\int_0^4 \int_0^{\sqrt{4-y}} \frac{xe^{2y}}{4-y} \, dx \, dy = \int_0^4 \left(\frac{1}{2} \frac{x^2 e^{2y}}{4-y}\right)\Big|_{x=0}^{x=\sqrt{4-y}} \, dy$$

$y = 4-x^2$
$x = \sqrt{4-y}$

$$= \int_0^4 \frac{1}{2} \frac{(4-y)e^{2y}}{4-y} \, dy = \frac{1}{2}\int_0^4 e^{2y} \, dy = \frac{1}{2}\left(\frac{1}{2}e^{2y}\right)\Big|_0^4$$

$$= \frac{1}{4}\left(e^8 - 1\right)$$