Name: __Key__

Calculus 251:C3   Quiz #13 - 6/23/2021   Topic: Polar coordinates

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.
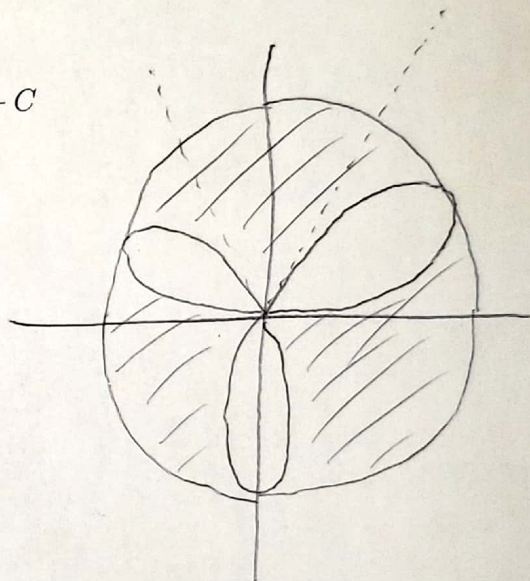
10 pts   (1) Let R be the region inside the circle $r = 3$ and outside the "rose" $r = 3\sin(3\theta)$.

(a) Sketch a graph of the two functions and shade the region R.

(b) Calculate the area of R.

Hint: $\int \sin^2(3x)\, dx = \dfrac{6x - \sin(6x)}{12} + C$

Area of Circle $= 9\pi$

(don't need calculus: $A = \pi r^2$)

Area of 1st quadrant petal

$$A = \frac{1}{2}\int_0^{\pi/3}(3\sin 3\theta)^2\, d\theta = \frac{9}{2}\int_0^{\pi/3}\sin^2 3\theta\, d\theta$$

$$= \frac{9}{2}\left(\frac{6\theta - \sin(6\theta)}{12}\right)\Big|_0^{\pi/3} = \frac{9}{2}\left(\frac{2\pi - 0}{12} - \frac{0-0}{12}\right) = \frac{9}{2}\cdot\frac{\pi}{6} = \frac{3\pi}{4}$$

Area of all 3 petals

$$3\left(\frac{3\pi}{4}\right) = \frac{9\pi}{4}$$

Area of R

$$9\pi - \frac{9\pi}{4} = \boxed{\frac{27\pi}{4}}$$