

Name: Key

Calculus 251:C3 Quiz #12 - 6/22/2021 Topic: Section 14.8

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts

- (1) Find the minimum and maximum values of the function $f(x, y) = 2xy$ subject to the constraint $5x^2 + 2y^2 = 50$, and indicate the points at which these values occur. **You must use the method of Lagrange multipliers to earn any credit on this quiz.**

$$\nabla f = \langle 2y, 2x \rangle$$

$$g(x, y) = 5x^2 + 2y^2 - 50$$

$$\nabla g = \langle 10x, 4y \rangle$$

$$\nabla f = \lambda \nabla g, \text{ so}$$

$$\left. \begin{aligned} 2y &= 10\lambda x \Rightarrow 2y^2 = 10\lambda xy \\ 2x &= 4\lambda y \Rightarrow 5x^2 = 10\lambda xy \end{aligned} \right\} \begin{array}{l} \text{multiply by } y \\ \text{multiply by } \frac{5}{2}x \end{array}$$

$$5x^2 + 2y^2 = 50 \quad \rightarrow \quad 2y^2 = 5x^2$$

$$5x^2 + 5x^2 = 50$$

$$x^2 = 5 \quad 2y^2 = 25$$

$$x = \pm\sqrt{5} \quad y = \pm\frac{5}{\sqrt{2}}$$

Candidate points	function values
$(\sqrt{5}, \frac{5}{\sqrt{2}})$	$2(\sqrt{5})(\frac{5}{\sqrt{2}}) = 5\sqrt{10}$
$(-\sqrt{5}, \frac{5}{\sqrt{2}})$	$-5\sqrt{10}$
$(\sqrt{5}, -\frac{5}{\sqrt{2}})$	$-5\sqrt{10}$
$(-\sqrt{5}, -\frac{5}{\sqrt{2}})$	$5\sqrt{10}$

Max $5\sqrt{10}$ at $(\sqrt{5}, \frac{5}{\sqrt{2}}), (-\sqrt{5}, -\frac{5}{\sqrt{2}})$

Min $-5\sqrt{10}$ at $(-\sqrt{5}, \frac{5}{\sqrt{2}}), (\sqrt{5}, -\frac{5}{\sqrt{2}})$