Name: Key

Calculus 251:C3 Quiz #11 - 6/21/2021 Topic: Section 14.7

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

(1) Let $f(x,y) = x^2 - xy - y^2 - 2x + 2y + 7$.

5 pts (a) Find the critical point of f and find the associated critical value. Then classify it as a local minimum, local maximum, or neither (saddle point).

$f_x = 2x - y - 2$ $\quad f_y = -x - 2y + 2$

So both are 0 when

$2x - y = 2$ and $x + 2y = 2$; add: $4x - 2y = 4$, $x + 2y = 2$

$5x = 6 \Rightarrow x = \frac{6}{5}$

$\frac{6}{5} + 2y = 2 \Rightarrow 2y = \frac{4}{5} \Rightarrow y = \frac{2}{5}$

CP $\left(\frac{6}{5}, \frac{2}{5}\right)$

$f_{xx} = 2$ $\quad f_{xy} = -1$ $\quad f_{yy} = -2$

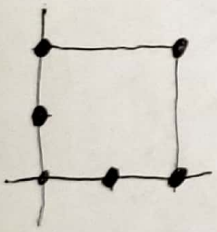
$D = \begin{vmatrix} 2 & -1 \\ -1 & -2 \end{vmatrix} = -5 < 0$, so saddle point

$f\left(\frac{6}{5}, \frac{2}{5}\right) = \frac{36}{25} - \frac{12}{25} - \frac{4}{25} - \frac{60}{25} + \frac{20}{25} + \frac{175}{25} = \frac{155}{25} = \frac{31}{5}$

CV $\frac{31}{5}$

5 pts (b) Let $\mathcal{S}$ be the square $\{(x,y) : 0 \le x \le 2, 0 \le y \le 2\}$. Find the minimum and maximum values of f on each of the four edges of $\mathcal{S}$. Then determine the global extreme values of f on $\mathcal{S}$.

You should use a table similar to the following to organize your work:



side of $\mathcal{S}$	Restriction of f	maximum	minimum
bottom $y=0$, $x \in [0,2]$	$x^2 - 2x + 7$	7	6
top $y=2$, $x \in [0,2]$	$x^2 - 4x + 7$	7	3
left $x=0$, $y \in [0,2]$	$-y^2 + 2y + 7$	8	7
right $x=2$, $y \in [0,2]$	$-y^2 + 7$	7	3

Bottom

$f(x) = x^2 - 2x + 7$

$f'(x) = 2x - 2$

CP at $x = 1$

$f(0) = 7$

$f(1) = 6$

$f(2) = 7$

Top

$f(x) = x^2 - 2x - 4 - 2x + 4 + 7$

$= x^2 - 4x + 7$

$f'(x) = 2x - 4$

CP at $x = 2$

$f(0) = 7$

$f(2) = 3$

Left

$f(y) = -y^2 + 2y + 7$

$f'(x) = -2y + 2$

CP at $y = 1$

$f(0) = 7$

$f(1) = 8$

$f(2) = 7$

Right

$f(y) = 4 - 2y - y^2 - 4 + 2y + 7$

$= -y^2 + 7$

$f'(y) = -2y$

CP at $y = 0$

$f(0) = 7$

$f(2) = 3$

Maximum of 8 at (0,1)

Minimum of 3 at (2,2)