

Name: Key

Calculus 251:C3 Quiz #10 - 6/17/2021 Topic: Section 14.6

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

- 4 pts (1) Find an equation of the plane tangent to the graph of $z = x^3y - xy^2$ at the point where $x = 2$ and $y = 1$.

$$f(2,1) = 8 - 2 = 6$$

$$f_x = 3x^2y - y^2 \quad f_x(2,1) = 12 - 1 = 11$$

$$f_y = x^3 - 2xy \quad f_y(2,1) = 8 - 4 = 4$$

Equation of tangent plane

$$z = 6 + 11(x - 2) + 4(y - 1)$$

- 6 pts (2) Use the linearization of an appropriate function to estimate the value of $\frac{\sqrt{4.1}}{2.9^2 + 1}$. Bonus point available if your answer is a (correct) single fraction in lowest terms.

$$f(x,y) = \frac{\sqrt{x}}{y^2 + 1} \quad P_0 = (4, 3)$$

$$f(4,3) = \frac{2}{10} = \frac{1}{5}$$

$$f_x = \frac{1}{2\sqrt{x}(y^2 + 1)} \quad f_x(4,3) = \frac{1}{2 \cdot 2 \cdot 10} = \frac{1}{40}$$

$$f_y = \frac{-2y\sqrt{x}}{(y^2 + 1)^2} \quad f_y(4,3) = \frac{-2 \cdot 3 \cdot 2}{100} = \frac{-3}{25}$$

$$L(x,y) = \frac{1}{5} + \frac{1}{40}(x - 4) - \frac{3}{25}(y - 3)$$

$$L(4.1, 2.9) = \frac{1}{5} + \frac{1}{40}\left(\frac{1}{10}\right) - \frac{3}{25}\left(-\frac{1}{10}\right)$$

$$= \frac{1}{5} + \frac{1}{400} + \frac{3}{250}$$

$$= \frac{400}{2000} + \frac{5}{2000} + \frac{24}{2000} = \frac{429}{2000}$$