Name: Key

Calculus 251:C3 Quiz #9 - 6/16/2021 Topic: Section 14.5

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

(1) Consider the function $f(x, y, z) = x^2 + 3y^2 - 2z^2$

3 pts (a) Find the gradient of f.

$\nabla f = \langle 2x, 6y, -4z \rangle$

1 pt (b) Evaluate the gradient of f at the point $P_0 = (3, 2, 1)$.

$\nabla f|_{P_0} = \langle 6, 12, -4 \rangle$

2 pts (c) Let $\vec{u} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$. Find $D_{\vec{u}}(P_0)$.

$\|\vec{u}\| = \sqrt{4+1+4} = 3 \qquad \hat{u} = \langle \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \rangle$

$D_{\hat{u}} f(P_0) = \langle 6, 12, -4 \rangle \cdot \langle \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \rangle = \frac{12+12+8}{3} = \frac{32}{3}$

For parts (d), (e), and (f), your answer does not need to be a unit vector.

1 pt (d) At P_0, what is the direction of greatest increase for f?

$\langle 6, 12, -4 \rangle$

1 pt (e) At P_0, what is the direction of greatest decrease for f?

$\langle -6, -12, 4 \rangle$

1 pt (f) At P_0, name **one** vector in a direction in which the value of f does not change?

$\langle 2, -1, 0 \rangle$ or any other vector $\perp$ to $\nabla f|_{P_0}$

1 pt (g) How many different correct answers are possible for the previous question?

infinitely many. There is an entire plane orthogonal to $\langle 6, 12, -4 \rangle$, and any vector in that plane works.