Name: __Key__

Calculus 251:C3   Quiz #6 - 6/9/2021   Topic: Sections 13.2-13.3

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. <u>Show and label all of your work.</u> Responses with no work may receive no credit even if the answer is correct.

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7 pts

(1) Solve the differential equation $\vec{r}''(t) = \langle 4, 0, -6t \rangle$ given the initial conditions $\vec{r}'(1) = \langle 4, 4, 4 \rangle$ and $\vec{r}(1) = \langle 5, 5, 5 \rangle$.

$$\vec{r}'(t) = \int \vec{r}''(t)\, dt = \langle 4t, 0, -3t^2 \rangle + \vec{C}_1$$

$$\langle 4,4,4 \rangle = \vec{r}'(1) = \langle 4, 0, -3 \rangle + \vec{C}_1 \implies \vec{C}_1 = \langle 0, 4, 7 \rangle$$

$$\vec{r}'(t) = \langle 4t, 4, 7 - 3t^2 \rangle$$

$$\vec{r}(t) = \int \vec{r}'(t)\, dt = \langle 2t^2, 4t, 7t - t^3 \rangle + \vec{C}_2$$

$$\langle 5,5,5 \rangle = \vec{r}(1) = \langle 2, 4, 6 \rangle + \vec{C}_2 \implies \vec{C}_2 = \langle 3, 1, -1 \rangle$$

$$\boxed{\vec{r}(t) = \langle 3+2t^2, 1+4t, -1+7t-t^3 \rangle}$$

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5 pts

(2) Find the length of the curve $\vec{r}(t) = \left\langle \frac{4}{3}t^{3/2}, \frac{1}{2}t^2, 2t \right\rangle$ from $t = 1$ to $t = 5$.

Hint: the answer is an integer!

Leaving out absolute value because $t+2 > 0$

$$\vec{r}'(t) = \langle 2\sqrt{t}, t, 2 \rangle$$

$$\| \vec{r}'(t) \| = \sqrt{(2\sqrt{t})^2 + t^2 + 2^2} = \sqrt{t^2 + 4t + 4} = \sqrt{(t+2)^2} = t + 2$$

$$S = \int_a^b \| \vec{r}'(t) \|\, dt = \int_1^5 (t+2)\, dt = \left[\frac{t^2}{2} + 2t \right]_{t=1}^{t=5} = \left(\frac{25}{2} + 10 \right) - \left(\frac{1}{2} + 2 \right)$$

$$= \frac{24}{2} + 8 = \boxed{20}$$