Calculus 251:C3   Quiz #5 - 6/8/2021   Topic: Sections 13.1-13.2

Instructions. Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

6 pts   (1) Find a parametrization of the line tangent to $\vec{r}(t) = \langle t^2, 2t^3, 6 - t \rangle$ at $t = 3$.

$$\vec{r}(3) = \langle 9, 54, 3 \rangle$$
$$\vec{r}'(t) = \langle 2t, 6t^2, -1 \rangle$$
$$\vec{r}'(3) = \langle 6, 54, -1 \rangle$$

tangent $L: \vec{x}(s) = \langle 9, 54, 3 \rangle + s \langle 6, 54, -1 \rangle$

$$\underline{\underline{or}}$$

$$\langle 9 + 6s, 54 + 54s, 3 - s \rangle$$

4 pts   (2) Find a parametrization of the intersection of the surfaces $x^2 + y^2 = 16$ and $z = xy^2$

$x^2 + y^2 = 16$ is a cylinder, so we can use

$$\vec{r}(t) = \langle 4\cos t, 4\sin t, z(t) \rangle$$

from $z = xy^2$, we get $z(t) = x(t) \, y(t)^2 = (4\cos t)(4\sin t)^2$
$$= 64 \cos t \sin^2 t$$

Thus, $\vec{r}(t) = \langle 4\cos t, 4\sin t, 64 \cos t \sin^2 t \rangle$, $t \in [0, 2\pi]$

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Alternatively, use $t = x$.
  1st equation: $\vec{r}_1(t) = \langle t, \sqrt{16 - t^2}, z(t) \rangle$, $\vec{r}_2(t) = \langle t, -\sqrt{16 - t^2}, z(t) \rangle$
  2nd equation $z(t) = xy^2 = t(\sqrt{16-t^2})^2 = 16t - t^3$

  So $\vec{r}_1(t) = \langle t, \sqrt{16 - t^2}, 16t - t^3 \rangle$, $\vec{r}_2(t) = \langle t, -\sqrt{16-t^2}, 16t - t^3 \rangle$, $t \in [-4, 4]$