

Name: Key

Calculus 251:C3 Quiz #4 - 6/7/2021 Topic: Section 12.5

**Instructions.** Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

10 pts (1) Determine whether the two lines parametrized below intersect.

$$\mathcal{L}_1: \vec{r}_1(t) = \langle 4 + 2t, 3 - t, 2 + 4t \rangle$$

$$\mathcal{L}_2: \vec{r}_2(t) = \langle 11 - 3t, -3 + 4t, 10t \rangle$$

If the lines intersect

- Find the point of intersection
- Find an equation of the plane containing both lines.

If the lines do not intersect

- Determine whether or not the lines are parallel
- Find the distance between the lines.

The lines intersect if  $\vec{r}_1(t) = \vec{r}_2(s)$  for some  $s, t \in \mathbb{R}$

$$\begin{cases} 4 + 2t = 11 - 3s \\ 3 - t = -3 + 4s \\ 2 + 4t = 10s \end{cases} \Rightarrow \begin{cases} 4 + 2t = 11 - 3s \\ 6 - 2t = -6 + 8s \end{cases} \Rightarrow \begin{cases} 10 = 5 + 5s \\ \Rightarrow s = 1 \end{cases}$$
  
$$\rightarrow 3 - t = -3 + 4(1) \Rightarrow -t = -2 \Rightarrow t = 2$$

$$\vec{r}_1(2) = \langle 4 + 4, 3 - 2, 2 + 8 \rangle = \langle 8, 1, 10 \rangle$$

Third equation:

$$2 + 4(2) = 10(1)$$

$$10 = 10 \quad \checkmark$$

check:  $\vec{r}_2(1) = \langle 11 - 3, -3 + 4, 10 \rangle = \langle 8, 1, 10 \rangle$

pt of intersection  $(8, 1, 10)$

Yes, they intersect

$$\vec{v}_1 = \langle 2, -1, 4 \rangle, \vec{v}_2 = \langle -3, 4, 10 \rangle$$

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ -3 & 4 & 10 \end{vmatrix} = -26\hat{i} - 32\hat{j} + 5\hat{k}, \text{ normal vector to plane}$$

(scalar multiples of this are also okay!)

$$\text{Eq. of plane: } -26(x - 8) - 32(y - 1) + 5(z - 10) = 0$$

(could use any point on either line - I chose pt. of intersection)