

Name: Key

Calculus 251:C3 Quiz #1 - 6/1/2021 Topic: Calculus II review

**Instructions.** Answer the questions in the spaces provided or on your own paper, then scan and upload to Canvas. Show and label all of your work. Responses with no work may receive no credit even if the answer is correct.

(1) Calculate each integral and fully simplify your answer.

3 pts

(a)  $\int_{-1}^2 x^2 e^{x^3} dx$

$u = x^3, du = 3x^2 dx$

Value of integral:  $\frac{1}{3} (e^8 - \frac{1}{e})$

$$= \frac{1}{3} \int_{-1}^2 3x^2 e^{x^3} dx = \frac{1}{3} \int_{-1}^8 e^u du = \frac{1}{3} e^u \Big|_{-1}^8 = \frac{1}{3} (e^8 - \frac{1}{e})$$

$u(-1) = -1$

$u(2) = 8$

3 pts

(b)  $\int_0^{\pi/4} (\cos \theta)^3 d\theta$

Value of integral:  $\frac{5\sqrt{2}}{12}$

$$= \int_0^{\pi/4} \cos \theta (1 - \sin^2 \theta) d\theta = \int_0^{\pi/4} (\cos \theta - \sin^2 \theta \cos \theta) d\theta$$

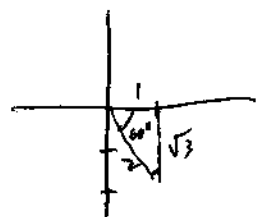
$$= (\sin \theta - \frac{1}{3} \sin^3 \theta) \Big|_0^{\pi/4} = \frac{\sqrt{2}}{2} - \frac{1}{3} \left(\frac{\sqrt{2}}{2}\right)^3 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{12} = \frac{5\sqrt{2}}{12}$$

1 pt (2) Convert  $(1, -\sqrt{3})$  from rectangular to polar coordinates.

$$(r, \theta) = \underline{\left(2, \frac{5\pi}{3}\right)}$$

$$r = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$$

Quadrant IV



so either  $-\frac{\pi}{3}$  or  $\frac{5\pi}{3}$

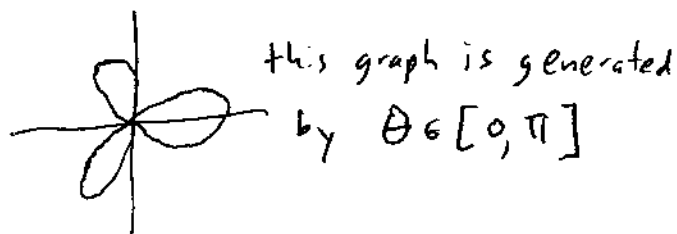
(or in fact  $\frac{5\pi}{3} + 2\pi n$  for any  $n \in \mathbb{Z}$ )

3 pts (3) Calculate the area enclosed by the graph of  $r = 2 \cos 3\theta$ .

Hint:  $\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$

Area enclosed:  $\underline{\pi}$

I admit that this question was mean. Sorry.



$$\begin{aligned} A &= \frac{1}{2} \int_0^{\pi} (2 \cos 3\theta)^2 d\theta = 2 \int_0^{\pi} \cos^2 3\theta d\theta \\ &= 2 \int_0^{\pi} \frac{1 + \cos 6\theta}{2} d\theta = \int_0^{\pi} (1 + \cos 6\theta) d\theta \\ &= \left( \theta + \frac{1}{6} \sin 6\theta \right) \Big|_0^{\pi} = \pi \end{aligned}$$