Calculus 251:C3 Worksheet 17.3

Note: This worksheet includes all of the recommended textbook problems with a few extras.

(1) Verify the Divergence Theorem for the given vector field and region.

(a) $\vec{F}(x,y,z) = \langle z, x, y\rangle$, the box $[0,4]\times[0,2]\times[0,3]$.

(b) $\vec{F}(x,y,z) = \langle y, x, z\rangle$, the region $x^2+y^2+z^2 \le 4$.

(c) $\vec{F}(x,y,z) = \langle 2x, 3z, 3y\rangle$, the region $x^2+y^2 \le 1, 0 \le z \le 2$.

(2) Use the Divergence Theorem to evaluate the flux $\iint_{\mathcal{S}} \vec{F}\cdot d\vec{S}$.

(a) $\vec{F}(x,y,z) = \langle xy^2, yz^2, zx^2\rangle$
$\mathcal{S}$ is the boundary of the cylinder $x^2+y^2 \le 4, 0 \le z \le 3$

(b) $\vec{F}(x,y,z) = \langle x^3, 0, z^3\rangle$
$\mathcal{S}$ is the boundary of the region $x^2+y^2+z^2 \le 4$, $x,y,z \ge 0$

(c) $\vec{F}(x,y,z) = \langle x, y^2, z+y\rangle$
$\mathcal{S}$ is the boundary of the region contained in the cylinder $x^2+y^2=4$ between the planes $z=x$ and $z=8$

(d) $\vec{F}(x,y,z) = \langle x^2-z^2, e^{z^2}-\cos x, y^3\rangle$
$\mathcal{S}$ is the boundary of the region bounded by $x+2y+4z=12$ and the coordinate planes in the first octant

(e) $\vec{F}(x,y,z) = \langle x+y, z, z-x\rangle$
$\mathcal{S}$ is the boundary of the region between the paraboloid $z = 9-x^2-y^2$ and the xy-plane