Difficulty guide for worksheet:	
C-level or B-level exam problem:	1, 2, 3, 5
A-level exam problem or challenge for extra study:	4, 6, 7
beyond the scope and/or removed from syllabus:	none

1. Let $\mathcal{C}$ be the piecewise linear path from $(1,0)$ to $(0,1)$ to $(-1,0)$ to $(1,0)$. Verify Green's Theorem for the line integral $\int_{\mathcal{C}} \big(xy\,dx + (x^2+x)\,dy\big)$.

2. Use Green's Theorem to calculate $\int_{\mathcal{C}} \big(y^2\,dx + x^2\,dy\big)$ where $\mathcal{C}$ is the triangular path from $(0,0)$ to $(0,1)$ to $(1,0)$.

3. Let $\mathcal{C}$ be the boundary of the region bounded $y = x^2$ and $x = y^2$. Assume that $\mathcal{C}$ is oriented anticlockwise. Calculate $\int_{\mathcal{C}} \big((xy+y^2)\,dx + (x-y)\,dy\big)$.

4. Consider the vector field $\boldsymbol{F} = \frac{1}{r}\boldsymbol{e}_r$.

(a) Let $\mathcal{C}$ be the circle of radius R centered at the origin, oriented anticlockwise. Evaluate the integral of $\boldsymbol{F}$ along $\mathcal{C}$ directly (no fancy theorems).

(b) Calculate $\boldsymbol{\nabla} \times \boldsymbol{F}$.

(c) In light of part (b), does $\boldsymbol{F}$ have a potential? If so, calculate a potential for $\boldsymbol{F}$.

(d) Recall that the integral of a conservative vector field along a closed curve is 0. Does this contradict your answers to parts (a), (b), and (c)?

(e) Repeat parts (a), (b), and (c) with the vector field $\boldsymbol{F} = \frac{1}{r^2}\langle -y,\ x\rangle$.

(f) Part (a) asks you to evaluate all line integrals in this problem directly without any fancy theorems. Now consider using Green's theorem to solve this problem. What can you say?

5. The velocity field of a certain fluid is given by $\boldsymbol{u} = \left\langle e^x \ln(y),\ x^2 + \frac{e^x}{y} \right\rangle$. Calculate the circulation of the fluid around the boundary of the region that is bounded above by $y = 3 - x$ and bounded below by $y = x^2 + 1$. Assume the boundary is oriented anticlockwise.

6. Let $\boldsymbol{F}(x,y) = \langle 2xe^y,\ x + x^2 e^y\rangle$ and let $\mathcal{C}$ be the path on the circle $x^2 + y^2 = 16$ from $P = (4,0)$ to $Q = (0,4)$. Let $I = \int_{\mathcal{C}} \boldsymbol{F} \cdot d\boldsymbol{r}$.

(a) Explain why direct evaluation of I is difficult.

(b) Show that $\boldsymbol{F}$ is not conservative but $\boldsymbol{G} = \boldsymbol{F} - \langle 0, x\rangle$ is conservative.

(c) Find a potential f for $\boldsymbol{G}$.

(d) Use parts (b) and (c) to calculate I.

Hint: Complete $\mathcal{C}$ into a closed curve and then use Green's Theorem and the gradient theorem.

7. Let $\mathcal{C}$ be the piecewise linear path from $(0,0)$ to $(2,2)$, to $(2,4)$ to $(0,6)$. (Note that $\mathcal{C}$ is not a closed curve.) Calculate $\int_{\mathcal{C}} \big((\sin(x) + y)\,dx + (3x+y)\,dy\big)$.

Hint: Save work by completing $\mathcal{C}$ into a closed curve and using Green's Theorem.