> **Difficulty guide for worksheet:**
> | | |
> |---|---|
> | C-level or B-level exam problem: | 1, 2, 3, 4, 5, 6 |
> | A-level exam problem or challenge for extra study: | 7, 8, 9 |
> | beyond the scope and/or removed from syllabus: | none |

1. Let $\mathcal{S}$ be the surface with parametrization $G(u, v) = (u^2 - v^2, \ u + v, \ u - v)$. Find an equation of the plane tangent to $\mathcal{S}$ at the point where $u = 2$ and $v = 3$.

2. Let $\mathcal{S}$ be the sphere centered at the origin with radius 3. Find an equation of the plane tangent to $\mathcal{S}$ at the point $(x, y, z) = (1, -2, 2)$.

3. Let $\mathcal{S}$ be the triangle with vertices $(0, 0, 3)$, $(1, 0, 2)$, and $(0, 4, 1)$.

    (a) Find a parametrization of $\mathcal{S}$.

    (b) Calculate $\displaystyle\iint_{\mathcal{S}} (xy + e^z) \ dS$.

4. Let $\mathcal{S}$ be the portion of the graph of $z = x + y^2$ above the triangle in the xy-plane with vertices $(0, 0, 0)$, $(1, 1, 0)$, and $(0, 1, 0)$. Calculate $\displaystyle\int_{\mathcal{S}} (z - x) \ dS$.

5. A surface is parametrized by $G(x, y) = (x, \ y, \ xy)$. Let $\mathcal{S}$ be the part of the surface with parameter domain $\mathcal{D} = \{(x, y) : x^2 + y^2 \le 1, \ x \ge 0 \ y \ge 0\}$.

    (a) Calculate $\boldsymbol{T}_x$, $\boldsymbol{T}_y$, and $\boldsymbol{N}(x, y)$.

    (b) Calculate the surface area of $\mathcal{S}$.

    (c) Calculate the average z-coordinate of $\mathcal{S}$.

6. Calculate the area of the portion of the plane $y + 2z = 2$ inside the cylinder $x^2 + y^2 = 1$.

7. Calculate the area of the lower portion cut from the sphere $x^2 + y^2 + z^2 = 2$ by the cone $z = \sqrt{x^2 + y^2}$.

8. Find the center of mass of a thin shell of density $\delta(x, y, z) = z^{-2}$ cut from the cone $z = \sqrt{x^2 + y^2}$ by the planes $z = 2$ and $z = 4$.

9. Let $\mathcal{S}$ be the surface that consists of the portion of the sphere $x^2 + y^2 + z^2 = 9$ in the first octant that is also both outside the cylinder $x^2 + y^2 = 1$ and above the plane $z = \sqrt{5}$.

    (a) Find a parametrization for $\mathcal{S}$.

    (b) Calculate the normal vector $\boldsymbol{N}$. Does the normal vector induced by your parametrization point inward or outward?