

Difficulty guide for worksheet:

C-level or B-level exam problem: 1, 2, 3, 5

A-level exam problem or challenge for extra study: 4

beyond the scope and/or removed from syllabus: none

1. For each part, calculate $\int_{\mathcal{C}} f \, ds$ for the given function f and curve \mathcal{C} .

(a) $f(x, y) = xy$; \mathcal{C} is the unit circle

(b) $f(x, y) = x^2 - 2y^2$; \mathcal{C} is the line segment from the origin to the point $(\sqrt{8}, \sqrt{8})$

(c) $f(x, y) = x$; \mathcal{C} is the curve $\mathbf{r}(t) = \langle t^3, 4t \rangle$ for $0 \leq t \leq 1$

(d) $f(x, y, z) = y - z$; \mathcal{C} is one turn of the helix $\mathbf{r}(t) = \langle 3 \cos(t), 3 \sin(t), 4t \rangle$ starting from the point $(3, 0, 0)$

(e) $f(x, y, z) = xz$; \mathcal{C} is the line segment from the origin to $(3, 2, 6)$ followed by the line segment from $(3, 2, 6)$ to $(7, 9, 10)$

2. For each part, calculate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ for the given vector field \mathbf{F} and curve \mathcal{C} .

(a) $\mathbf{F}(x, y) = \langle x, y \rangle$; \mathcal{C} is the portion of the parabola $16y = x^2$ from $(4, 1)$ to the origin.

(b) $\mathbf{F}(x, y) = \frac{\langle x, y \rangle}{(x^2 + y^2)^{3/2}}$; \mathcal{C} is the path $\mathbf{r}(t) = \langle t^2, 3t^2 \rangle$ for $1 \leq t \leq 2$

(c) $\mathbf{F}(x, y, z) = \langle -y, x, z \rangle$; \mathcal{C} is the helix $\mathbf{r}(t) = \langle 2 \cos(t), 2 \sin(t), \frac{t}{2\pi} \rangle$ for $0 \leq t \leq 2\pi$

(d) $\mathbf{F}(x, y, z) = \langle x + y, x - y, x \rangle$; \mathcal{C} is the line segment from $(1, 2, 4)$ to $(3, 8, 13)$

(e) $\mathbf{F}(x, y) = \langle x^2, xy \rangle$; \mathcal{C} is a quarter circle with radius 3 centered at the origin from $(0, 3)$ to $(-3, 0)$

3. Let \mathcal{C} be the path $\mathbf{r}(t) = \langle \cos(t), \tan(t), t \rangle$ for $0 \leq t \leq \pi/4$. Calculate

$$\int_{\mathcal{C}} (z \, dx + x^2 \, dy + y \, dz)$$

4. A particle travels in the force field $\mathbf{F}(x, y, z) = \langle e^z, e^{x-y}, e^y \rangle$ along the piecewise linear path starting from the origin, then to $(0, 0, 1)$, then to $(0, 1, 1)$, and ending at $(-1, 1, 1)$. Calculate the work done by the force field on the particle.

5. Let \mathcal{C} be the curve $y = x^{4/3}$ in the xy -plane from $(1, 1)$ to $(8, 16)$. Let \mathcal{R} be the sheet that consists of all points below the surface $z = x/y$ and above the curve \mathcal{C} . Calculate the area of \mathcal{R} .