Difficulty guide for worksheet:	
C-level or B-level exam problem:	1, 2a, 2b, 2c, 2d, 2e, 2f, 2g, 2h, 2i
A-level exam problem or challenge for extra study:	2j, 2k, 3
beyond the scope and/or removed from syllabus:	none

1. Identify which of the following expressions makes sense and for those that do, say whether it is a vector or a scalar. Assume f and g are sufficiently differentiable scalar-valued functions, and assume $\boldsymbol{F}$ and $\boldsymbol{G}$ are sufficiently differentiable vector fields. The vectors $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are constants.

(a) $(\boldsymbol{a} \cdot \boldsymbol{b}) \times \boldsymbol{c}$

(b) $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c})$

(c) $\operatorname{grad}(\boldsymbol{F})$

(d) $\boldsymbol{F} \cdot \operatorname{grad}(f)$

(e) $\operatorname{div}(\boldsymbol{F} \cdot \boldsymbol{G})$

(f) $\operatorname{curl}(\boldsymbol{F} \times \boldsymbol{G})$

(g) $\boldsymbol{F} \times \operatorname{grad}(f)$

(h) $\operatorname{curl}(f\boldsymbol{F})$

(i) $\operatorname{grad}(fg)$

(j) $\operatorname{curl}(\boldsymbol{F} \cdot \boldsymbol{G})$

(k) $\operatorname{div}(f\boldsymbol{F})$

(l) $\operatorname{div}(\boldsymbol{F}\boldsymbol{G})$

(m) $\operatorname{curl}(\operatorname{div}(\boldsymbol{F}))$

(n) $\operatorname{grad}(\operatorname{div}(\boldsymbol{F}))$

(o) $\operatorname{curl}(\operatorname{curl}(\boldsymbol{F}))$

2. Calculate $\operatorname{curl}(\boldsymbol{F})$ and $\operatorname{div}(\boldsymbol{F})$ for each vector field $\boldsymbol{F}$. Also, for each $\boldsymbol{F}$, find a potential function f or show that no such potential exists.

(a) $\boldsymbol{F} = \langle x, y \rangle$

(b) $\boldsymbol{F} = \langle y, x, x - y \rangle$

(c) $\boldsymbol{F} = \langle y + z, x + z, x + y \rangle$

(d) $\boldsymbol{F} = \langle -y, -x \rangle$

(e) $\boldsymbol{F} = \langle -y, x \rangle$

(f) $\boldsymbol{F} = \langle e^z, e^z, e^z(x - y) \rangle$

(g) $\boldsymbol{F} = \left\langle x^3, 3y, -z^3 \right\rangle$

(h) $\boldsymbol{F} = \langle ye^{xy}, xe^{xy} \rangle$

(i) $\boldsymbol{F} = \left\langle yz^2, xz^2, 2xyz \right\rangle$

(j) $\boldsymbol{F} = \dfrac{\langle x, y, z \rangle}{x^2 + y^2 + z^2}$

(k) $\boldsymbol{F} = \dfrac{\langle x, y, z \rangle}{\sqrt{x^2 + y^2 + z^2}}$

3. Determine whether the following statements are true or false. If the statement is true, explain why. If the statement is false, give a counterexample.

Recall: It is well known that for a function f of a single variable, if $f'(x) = 0$ for all x, then f is a constant function.

(a) If $\nabla \cdot \boldsymbol{F} = 0$ for all points (x, y, z), then $\boldsymbol{F}$ is constant.

(b) If $\nabla \times \boldsymbol{F} = \mathbf{0}$ for all points (x, y, z), then $\boldsymbol{F}$ is constant.

(c) If $\nabla \cdot \boldsymbol{F} = 0$ and $\nabla \times \boldsymbol{F} = \mathbf{0}$ for all points (x, y, z), then $\boldsymbol{F}$ is constant.

(d) A vector field consisting of parallel vectors has zero curl.

(e) A vector field consisting of parallel vectors has zero divergence.

(f) $\nabla \times \boldsymbol{F}$ is orthogonal to $\boldsymbol{F}$.