Difficulty guide for worksheet:
C-level or B-level exam problem: 1
A-level exam problem or challenge for extra study: 2, 3, 4
beyond the scope and/or removed from syllabus: none

1. Let $\mathcal{D}$ be the parallelogram in the xy-plane with vertices $(0,0)$, $(-2,5)$, $(1,7)$, and $(-1,12)$.

(a) Find a linear mapping G that maps $[0,1]\times[0,1]$ in the uv-plane onto $\mathcal{D}$.

(b) Use a change of variables to evaluate $\iint_{\mathcal{D}} y^2\,dA$.

2. Let $G(u,v) = \left(\dfrac{u}{v+1}, \dfrac{uv}{v+1}\right)$.

(a) Describe the image, in the xy-plane, of the vertical line $u = c$.

(b) Describe the image, in the xy-plane, of the horizontal line $v = c$.

(c) Calculate $\mathrm{Jac}(G)$ as a function of u and v.

(d) Calculate $G^{-1}(x,y)$.

(e) Let $\mathcal{D}$ be the region in the xy-plane bounded by the lines $x+y=3$, $x+y=6$, $y=x$, and $y=2x$. Find a rectangle $\mathcal{R}$ in the uv-plane such that $G(\mathcal{R}) = \mathcal{D}$.

(f) Use the mapping G to calculate the integral $\iint_{\mathcal{D}} (x+y)\,dA$.

3. Let $G(u,v) = (u - uv, uv)$.

(a) Describe the image, in the xy-plane, of the vertical line $u = c$.

(b) Describe the image, in the xy-plane, of the horizontal line $v = c$. (Be careful to consider the the case $c = 1$ separately. Why?)

(c) Compute the Jacobian of G.

(d) Let $\mathcal{D}$ be the quadrilateral in the xy-plane with vertices $(a,0)$, $(b,0)$, $(0,a)$, and $(0,b)$ with $0 < a < b$. Find a rectangle $\mathcal{R}$ in the uv-plane such that $G(\mathcal{R}) = \mathcal{D}$.

(e) Elementary geometry shows that the area of $\mathcal{D}$ is $\frac{1}{2}(b^2 - a^2)$. Use the mapping G and an appropriate integral to verify this formula.

(f) Use the mapping G to calculate $\iint_{\mathcal{D}} xy\,dA$.

4. Consider the mapping $G(u,v) = \left(u^2 - v^2, 2uv\right)$. Let $\mathcal{T}$ be the triangular region in the uv-plane given by $0 \le v \le u \le 2$, and put $\mathcal{D} = G(\mathcal{T})$.

(a) Sketch the region $\mathcal{D}$ in the xy-plane. What is the image, in the xy-plane, of each boundary curve of $\mathcal{T}$?

(b) Use the mapping G to calculate $\iint_{\mathcal{D}} \sqrt{x^2+y^2}\,dA$.