

Difficulty guide for worksheet:

C-level or B-level exam problem: 1a, 1b, 1c, 1e, 2a, 3, 4, 5

A-level exam problem or challenge for extra study: 1d, 1f, 2b, 6

beyond the scope and/or removed from syllabus: none

1. Find the volume of each region or type of solid.

- the region bounded by the paraboloid $z = x^2 + y^2$ and the cone $z = 2 - \sqrt{x^2 + y^2}$
- the region below the surface $z = xy + 10$ and above the annular region $\mathcal{R} = \{(x, y) : 4 \leq x^2 + y^2 \leq 16\}$
- the region inside both the cone $\varphi = \frac{\pi}{6}$ and the sphere $\rho = 4$
- a spherical cap of radius R and height H
- the solid obtained from a sphere centered at the origin with radius 2 after a cylindrical hole of radius 1 is drilled through the center of the sphere perpendicular to its base
- the solid bounded by the cylinder $x^2 + y^2 = 1$, the xy -plane, and the plane $z = x + y$

2. Let \mathcal{D}_1 be the disk in the xy -plane centered at the origin with radius 2. Let \mathcal{D}_2 be the disk in the xy -plane centered at $(2, 0)$ with radius 2. Suppose $g(x, y)$ is continuous for all x and y . Write an iterated integral in polar coordinates for $\iint_{\mathcal{R}} g(x, y) dA$, where \mathcal{R} is...

- ...the region outside \mathcal{D}_1 and inside \mathcal{D}_2 .
- ...the region inside both \mathcal{D}_1 and \mathcal{D}_2 .

3. Calculate $\int_0^5 \int_0^y x dx dy$ by changing to polar coordinates.

4. Calculate $\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \int_0^{9-3\sqrt{x^2+y^2}} dz dx dy$ by changing to cylindrical coordinates.

5. Let \mathcal{W} be the solid region bounded above by the plane $z = 5$ and bounded below by the cone $z^2 = x^2 + y^2$. Use spherical coordinates to calculate $\iiint_{\mathcal{W}} \sqrt{x^2 + y^2 + z^2} dV$.

6. Let \mathcal{W} be the region within the cylinder $x^2 + y^2 = 2$ between the xy -plane and the cone $z = \sqrt{x^2 + y^2}$. Calculate the integral of $f(x, y) = x^2 + y^2$ over \mathcal{W} using...

- ...rectangular coordinates.
- ...cylindrical coordinates.
- ...spherical coordinates.

Which of these do you think is easiest?