

Difficulty guide for worksheet:

C-level or B-level exam problem: 1a, 1b, 1c, 1d, 1f, 1h, 1j, 2b, 3a

A-level exam problem or challenge for extra study: 1e, 1g, 1i, 2a

beyond the scope and/or removed from syllabus: 1k, 1l, 3b

1. Calculate $\int_{\mathcal{D}} y \, dA$, where \mathcal{D} is the domain

$$\mathcal{D} = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, x^2 \leq y \leq 4 - x^2\}$$

2. Let \mathcal{T} be the triangle in the xy -plane with vertices $(3, 0)$, $(0, 6)$, and $(3, 6)$. Calculate $\int_{\mathcal{T}} xy \, dA$.
3. Let \mathcal{T} be the trapezoid in the xy -plane with vertices $(0, 0)$, $(4, 0)$, $(4, 2)$, and $(2, 2)$. Suppose $f(x, y)$ is continuous on \mathcal{T} . Write $\int_{\mathcal{T}} f(x, y) \, dA$ as an iterated integral, or sum of iterated integrals, in each possible order ($dydx$ and $dxdy$).
4. Let \mathcal{R} be the region in the xy -plane bounded by $x^2 + y^2 = 4$ and $x = 1$. Calculate $\int_{\mathcal{R}} \frac{y}{x} \, dA$.
5. For each integral, sketch the domain of integration and express as an iterated integral in the opposite order. For parts (c) and (d), also evaluate the integral.

(a) $\int_0^8 \int_x^8 f(x, y) \, dydx$

(c) $\int_0^1 \int_y^1 \frac{\sin(x)}{x} \, dxdy$

(b) $\int_0^1 \int_{e^x}^e f(x, y) \, dydx$

(d) $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3 + 1} \, dxdy$

6. Let \mathcal{R} be the region in the xy -plane bounded by $y = e^x$ and $y = e^{\sqrt{x}}$. Calculate the volume of the solid region below the graph of $z = \frac{1}{\ln(y)}$ and above \mathcal{R} .