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| ***Difficulty guide for worksheet:*** | |
| *C-level or B-level exam problem*: | 1, 3, 4, 5 |
| *A-level exam problem or challenge for extra study*: | 2, 6 |
| *beyond the scope and/or removed from syllabus*: | none |

**1.** For each function, find all critical points and classify each of them as a local minimum, local maximum, or neither (saddle).

(a) $f(x, y) = 3x^2 - 4y^2$

(b) $f(x, y) = x^3 + 6xy - 6x + y^2 - 2y$

(c) $f(x, y) = ye^x - e^y$

(d) $f(x, y) = x^3y + 12x^2 - 8y$

**2.** Let $f(x, y) = y^2x - yx^2 + xy$.

(a) Show that the critical points $(x, y)$ satisfy the equations

$$\begin{aligned} y(y - 2x + 1) &= 0 \\ x(2y - x + 1) &= 0 \end{aligned}$$

(b) Show that $f$ has four critical points.

*Hint:* For three of these points, at least one of $x$ and $y$ is 0. For the fourth point, both $x$ and $y$ are nonzero.

(c) Classify each critical point as either a local minimum, local maximum, or neither (saddle).

**3.** Let $f(x, y) = x^2 + y^2 - 2y + 1$ and let $\mathcal{S}$ be the square $\{(x, y) : -1 \leq x \leq 1, -1 \leq y \leq 1\}$.

(a) Find the critical point(s) of $f$ and find the associated critical value(s). Then classify each critical point as a local minimum, local maximum, or neither (saddle).

(b) Find the minimum and maximum values of $f$ on each of the four edges of $\mathcal{S}$. Then determine the global extreme values of $f$ on $\mathcal{S}$. Fill in the table below as you work.

| edge of $\mathcal{S}$ | bottom edge | right edge | top edge | left edge |
|---|---|---|---|---|
| minimum value of $f$ | | | | |
| maximum value of $f$ | | | | |

**4.** Let $f(x, y) = x^2 + y^2 - 2x - 2y$ and let $\mathcal{T}$ be the closed region bounded by the triangle with vertices $(0, 0)$, $(2, 0)$, and $(0, 2)$.

(a) Find the critical point(s) of $f$ and find the associated critical value(s). Then classify each critical point as a local minimum, local maximum, or neither (saddle).

(b) Find the minimum and maximum values of $f$ on each of the three edges of $\mathcal{T}$. Then determine the global extreme values of $f$ on $\mathcal{T}$. Fill in the table below as you work.

| edge of $\mathcal{T}$ | bottom edge | left edge | slant edge |
|---|---|---|---|
| minimum value of $f$ | | | |
| maximum value of $f$ | | | |

**5.** Let $f(x,y) = 2x^2 + y^2$ and let $\mathcal{D}$ be the closed disk $\{(x,y) : x^2 + y^2 \leq 4\}$.

(a) Find the critical point(s) of $f$ and find the associated critical value(s). Then classify each critical point as a local minimum, local maximum, or neither (saddle).

(b) Find the minimum and maximum values of $f$ on the boundary of $\mathcal{D}$. Then determine the global extreme values of $f$ on $\mathcal{D}$.

**6.** Let $f(x,y) = \dfrac{2y^2 - x^2}{2 + 2x^2 y}$ and let $\mathcal{R}$ be the closed region bounded by the lines $y = x$, $y = 2x$, and $y = 2$.

(a) Find the critical point(s) of $f$ and find the associated critical value(s). Then classify each critical point as a local minimum, local maximum, or neither (saddle).

(b) Find the minimum and maximum values of $f$ on each of the three edges of $\mathcal{R}$. Then determine the global extreme values of $f$ on $\mathcal{R}$. Fill in the table below as you work.

| edge of $\mathcal{R}$ | left edge | right edge | top edge |
|---|---|---|---|
| minimum value of $f$ | | | |
| maximum value of $f$ | | | |