Difficulty guide for worksheet:	
C-level or B-level exam problem:	3, 4, 5, 6
A-level exam problem or challenge for extra study:	1
beyond the scope and/or removed from syllabus:	2

1. Let r, s, and t be independent parameters and suppose x, y, and z are given by

$$x = 4r - s - 6t$$
$$y = -r + 2s + 5t$$
$$z = 7r + 3s - t$$

Let $w = f(x, y, z)$ where f is an arbitrary differentiable function. Calculate the sum

$$A(r, s, t) = \frac{\partial w}{\partial r} - 2\frac{\partial w}{\partial s} + \frac{\partial w}{\partial t}$$

Write your answer as a function of r, s, and t. Simplify as much as possible.
Since f is arbitrary, your answer may still contain the symbol f or related symbols. But you must write your answer as a function of r, s, and t.

2. Let $z = f(x, y)$, where f is an arbitrary differentiable function. Suppose $x = r\cos(\theta)$ and $y = r\sin(\theta)$. Calculate both z_r and z_θ. Express your answer in terms of r and θ only.
This exercise shows how the derivatives in polar coordinates are related to the derivatives in rectangular coordinates.

3. Use the multivariable chain rule to calculate the specified partial derivatives.

(a) z_s and z_t, where $z = x^2 \sin(y)$, $x = s - t$, and $y = t^2$

(b) w_s and w_t, where $w = \dfrac{x - z}{y + z}$, $x = s + t$, $y = st$, and $z = s - t$

(c) $\dfrac{dU}{dt}$, where $U = \frac{xy^2}{z^8}$, $x = e^t$, $y = \sin(3t)$, and $z = 4t + 1$

4. Suppose x and y are implicitly related by the equation $F(x, y) = 0$. Use the multivariable chain rule to show

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

Then use this result to find $y'(x)$ if $x^2 = xy^2 + \sin(y) + 3$.

5. Suppose x, y, and z are implicitly related by the equation $F(x, y, z) = 0$. Use the multivariable chain rule to show

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \quad , \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Then use this result to find z_x and z_y if $x^2yz^2 = 10 - 3xz^3$.

6. For each implicit equation, calculate the specified partial derivative.

(a) z_x if $\sqrt{x^2 + 2xz + z^4} = 3$

(b) y_z if $y\ln(x^2 + y^2 + 4z) = 1$