

Difficulty guide for worksheet:

C-level or B-level exam problem: 1, 2, 3, 5a

A-level exam problem or challenge for extra study: 4, 5b

beyond the scope and/or removed from syllabus: none

- Find the rate of change of f at the point P in the direction \mathbf{u} .
 - $f(x, y) = \sin(xy + y^2)$, $P = (0, \sqrt{\pi})$, $\mathbf{u} = \langle 2, -3 \rangle$
 - $f(x, y) = \sqrt{1 + x^2y + xy^2}$, $P = (3, 4)$, \mathbf{u} is in direction toward origin
 - $f(x, y) = \frac{xy}{1 + xy^3}$, $P = (-1, 2)$, \mathbf{u} is in direction 30 degrees south of west
 - $f(x, y, z) = xyz^2 + x^3z$, $P = (1, -1, 1)$, $\mathbf{u} = \langle 0, 1, 1 \rangle$
- The temperature at the point (x, y, z) is given by $T(x, y, z) = xyz^{-1}$. A particle travels on the path given by $\mathbf{r}(t) = \langle e^t, t, t^2 \rangle$. Find the rate of change of the temperature along the particle's path at $t = 1$.
- Suppose that $\nabla f_P = \langle 2, -4, 4 \rangle$. Is f increasing or decreasing at P in the direction $\mathbf{v} = \langle 2, 1, 3 \rangle$?
- The height of some terrain is modeled by the equation $h = xe^{x^2-y}$. You are currently standing on the terrain above the point $P = (1, 1)$. Assume the positive y -axis points in the north direction.
 - If you travel from P in the northerly direction, what angle of inclination do you measure?
 - Suppose you decide to travel from P in the direction of steepest descent. Find the compass angle along which you should initially travel. (Assume the angle is measured anticlockwise from the positive x -axis.)
 - What is the steepest possible angle of inclination at P ?
 - Your friend, who also starts at P , does not want to exert himself, and so he decides to walk along the terrain, keeping a constant height. Find the compass angle along which your friend should initially travel from P . (Assume the angle is measured anticlockwise from the positive x -axis.)
- Find a function f with the given gradient.
 - $\nabla f = \langle y^2 \sin(xy^2) + 6x, 2xy \sin(xy^2) \rangle$
 - $\nabla f = \langle 2xyz^2, x^2z^2 + 8yz^3, 2x^2yz + 12y^2z^2 \rangle$