Difficulty guide for worksheet:	
C-level or B-level exam problem:	1, 3, 4, 5, 6
A-level exam problem or challenge for extra study:	2
beyond the scope and/or removed from syllabus:	none

1. Calculate all first derivatives for each function f.

(a) $f(x,y) = \cos\left(\dfrac{y}{x+y}\right)$

(b) $f(u,v) = \ln(u^2 + uv)$

(c) $f(x,y,z) = ze^{xz-x^2z^3}$

(d) $f(s,t) = \tan^{-1}(st^2)$

2. Calculate f_{xyxzy} for the following function.

$$f(x,y,z) = y\sin(xz)\sin(x+z) + (x+z^2)\tan(y) + x\tan\left(\frac{z+z^{-1}}{y-y^{-1}}\right)$$

3. Prove that there is no function f such that $f_x = xy^2$ and $f_y = -x^2y$.

4. Find an equation of the plane tangent to the graph of f at the indicated point.

(a) $f(x,y) = 3x^2y - x^3y^2$ at $(-1,1)$

(b) $f(x,y) = ye^{x/y}$ at $(\ln(2),2)$

5. Use a linear approximation to estimate the value of $\sqrt{\dfrac{9.2}{3.9}}$.

6. Let $f(x,y) = 3x^2 - xy - y^2 - 18x$. Find all points on the graph of f where the tangent plane is parallel to the indicated plane.

(a) the xy-plane

(b) the plane $2x - 5x + 2z = 1$

Solutions:

① (a) $f_x = -\sin\left(\frac{y}{x+y}\right) \cdot \frac{-y}{(x+y)^2}$

$f_y = -\sin\left(\frac{y}{x+y}\right) \cdot \frac{x}{(x+y)^2}$

(b) $f_u = \frac{2u+v}{u^2+uv}$

$f_v = \frac{u}{u^2+uv}$

(c) $f_x = ze^{xz-x^2z^3} \cdot (z-2xz^3)$

$f_y = 0$

$f_z = ze^{xz-x^2z^3} \cdot (x-3x^2z^2) + e^{xz-x^2z^3}$

(d) $f_s = \frac{t^2}{1+s^2t^4}$

$f_t = \frac{2st}{1+s^2t^4}$

② $f_{xyxzy} = 0$

③ If f existed, we would have

$f_{xy} = f_{yx}$, but

$$f_{xy} = (f_x)_y = 2xy$$
$$f_{yx} = (f_y)_x = -2xy$$

> not equal

④ (a) $f_x(-1,1) = (6xy - 3x^2y^2)\big|_{(-1,1)}$

$= -6 - 3 = -9$

$f_y(-1,1) = (3x^2 - 2x^3y)\big|_{(-1,1)}$

$= 3 + 2 = 5$

$f(-1,1) = 3 + 1 = 4$

eq. of plane: $z = 4 - 9(x+1) + 5(y-1)$

(b) $f_x(\ln 2, 2) = e^{x/y}\big|_{(\ln 2, 2)} = \sqrt{2}$

$f_y(\ln 2, 2) = e^{x/y}\left(1 - \frac{x}{y}\right)\Big|_{(\ln 2, 2)} = \sqrt{2}\left(1 - \frac{\ln 2}{2}\right)$

$f(\ln 2, 2) = 2\sqrt{2}$

eq. of plane:

$$z = 2\sqrt{2} + \sqrt{2}(x - \ln(2)) + \sqrt{2}\left(1 - \frac{\ln 2}{2}\right)(y-2)$$

(5) $f(x,y) = \sqrt{\frac{x}{y}}$ @ $(x,y) = (9,4)$

$$f_x(9,4) = \frac{1}{2\sqrt{xy}}\Big|_{(9,4)} = \frac{1}{12}$$

$$f_y(9,4) = -\frac{1}{2}\sqrt{\frac{x}{y^3}}\Big|_{(9,4)} = -\frac{3}{16}$$

$$f(9.2, 3.9) \approx f(9,4) + \frac{1}{12}(9.2-9) - \frac{3}{16}(3.9-4)$$

$$= \frac{3}{2} + \frac{1}{12}\left(\frac{2}{10}\right) - \frac{3}{16}\left(-\frac{1}{10}\right)$$

$$= \frac{3}{2} + \frac{1}{60} + \frac{3}{160}$$

(6) $f_x = 6x - y - 18$

$f_y = -x - 2y$

(a) $f_x = f_y = 0 \Rightarrow x = \frac{36}{13}, y = -\frac{18}{13}$

$$z = f\left(\frac{36}{13}, -\frac{18}{13}\right)$$

(b) eq. of plane is:

$$z = \frac{1}{2} - x + \frac{5}{2}y$$

$$\left.\begin{aligned} f_x &= 6x - y - 18 = -1 \\ f_y &= -x - 2y = \frac{5}{2} \end{aligned}\right\} \Rightarrow x = \frac{63}{26},\ y = -\frac{64}{26}$$