Difficulty guide for worksheet:

C-level or B-level exam problem:	1, 2, 4, 6
A-level exam problem or challenge for extra study:	5
beyond the scope and/or removed from syllabus:	3

1. Which of the following does *not* parametrize a line or some portion of a line? Explain your answer.

(a) $\boldsymbol{r}(t) = \langle 2+3t,\ 9-t,\ 12+7t\rangle$

(b) $\boldsymbol{r}(t) = \langle 1-t^2,\ 3+3t^2,\ t^3\rangle$

(c) $\boldsymbol{r}(t) = \langle 2\cos(2t),\ 5+3\cos(2t),\ \sin(2t)\rangle$

(d) $\boldsymbol{r}(t) = \langle t^3,\ 4-8t^3,\ 8+3t^3\rangle$

2. Find a parametrization of each described curve.

(a) A circle of radius 3 with center $(-2, 3, 1)$, lying in a plane parallel to the xz-plane.

(b) The ellipse $4y^2 + 9z^2 = 36$ translated to have center $(-1, 10, -5)$.

(c) The intersection of the surfaces $y^2 - z^2 = x - 2$ and $y^2 + z^2 = 9$.

(d) The intersection of the sphere $x^2 + y^2 + z^2 = 1$ and the paraboloid $z = x^2 + y^2$.

3. We will show that the curve with the following parametrization lies in a plane.

$$\boldsymbol{r}(t) = \langle t^2 - 1,\ t - 2t^2,\ 4 - 6t\rangle$$

(a) Show that the points on the curve at $t = 0$, $t = 1$, and $t = 2$ do not lie on a line. Then find an equation of the plane that they determine.

(b) Show that for all t, the points on $\mathcal{C}$ satisfy the equation of the plane from part (a).

4. Find a parametrization of the line tangent to the curve at the indicated value of t.

(a) $\boldsymbol{r}(t) = \langle 1-t^2,\ 5t,\ 2t^3\rangle$ at $t = 2$

(b) $\boldsymbol{r}(t) = 4t^{-1}\boldsymbol{i} - 6t^{-3}\boldsymbol{k}$ at $t = 1$

5. For $0 \le t \le 4\pi$, the path of a particle is parametrized by

$$\boldsymbol{r}(t) = \langle \cos(t)\sin(t),\ \sin(t)^2,\ \sin(t)\rangle$$

(a) Show that the path of the particle is a closed loop.

(b) Let $\mathcal{C}$ be the *curve* on which the particle travels. How many times does the particle traverse $\mathcal{C}$ from $t = 0$ to $t = 4\pi$? Justify your answer.

(c) Find an integral whose value is the length of $\mathcal{C}$.

(d) Find an integral whose value is the distance traveled by the particle.

6. Calculate the length of the described curve.

(a) $\boldsymbol{r}(t) = \langle 4t,\ \sqrt{3}t^{3/2},\ t^{3/2}\rangle$, $0 \le t \le 1$

(b) $\boldsymbol{r}(t) = \langle 2t,\ \ln(t),\ t^2\rangle$, $1 \le t \le 4$

Solutions:

① (a) Yes. (b) No. (c) No. (d) Yes.

② (a) $\vec{r}(t) = \langle -2+3\cos(t),\ 3,\ 1+3\sin(t) \rangle$

$0 \le t < 2\pi$

(b) $\vec{r}(t) = \langle -1,\ 10+3\cos(t),\ -5+2\sin(t) \rangle$

$0 \le t < 2\pi$

(c) $\vec{r}(t) = \langle 2+9\cos(2t),\ 3\cos(t),\ 3\sin(t) \rangle$

$0 \le t < 2\pi$

(d) Let $R = \dfrac{-1+\sqrt{5}}{2}$.

$\vec{r}(t) = \left\langle \sqrt{1-R^2}\cos(t),\ \sqrt{1-R^2}\sin(t),\ R \right\rangle$

$0 \le t < 2\pi$

③ (a) $\vec{u} = \vec{r}_1(1) - \vec{r}_1(0) = \langle 1, -1, -6 \rangle$

$\vec{v} = \vec{r}_1(2) - \vec{r}_1(0) = \langle 4, -6, -12 \rangle$

$\vec{u} \times \vec{v} = \langle -24, -12, -2 \rangle$

Since $\vec{u} \times \vec{v} \neq 0$, the three points are not collinear.

$\vec{n} = \langle -24, -12, -2 \rangle$

$P = \vec{r}_1(0) = (-1, 0, 4)$

eq. of plane: $-24(x+1) - 12y - 2(z-4) = 0$

(b) $-24(t^2-1+1) - 12(t-2t^2) - 2(4-6t-4) =$

$= -24t^2 - 12t + 24t^2 + 12t = 0$ ✓

4 (a) $\vec{v} = \vec{r}\,'(2) = \langle -2t, 5, 6t^2 \rangle \big|_{t=2}$

$= \langle -4, 5, 24 \rangle$

$\vec{r}(2) = \langle -3, 10, 16 \rangle$

$\vec{L}(t) = \langle -3-4t, 10+5t, 16+24t \rangle$

(b) $\vec{v} = \vec{r}\,'(1) = \langle -4t^{-2}, 0, 18t^{-4} \rangle \big|_{t=1}$

$= \langle -4, 0, 18 \rangle$

$\vec{r}(1) = \langle 4, 0, -6 \rangle$

$\vec{L}(t) = \langle 4-4t, 0, -6+18t \rangle$

5 (a) $\vec{r}(t)$ is continuous and $\vec{r}(0) = \vec{r}(4\pi)$.

(b) twice

(c) $\int_0^{2\pi} \sqrt{\frac{1}{2}(3+\cos(2t))}\, dt$

(d) $\int_0^{4\pi} \sqrt{\frac{1}{2}(3+\cos(2t))}\, dt$

6 (a) $\int_0^1 \sqrt{16+9t}\, dt = \frac{2}{27}(16+9t)^{3/2}\Big|_0^1$

$= \frac{2}{27}(125-64) = \frac{122}{27}$

(b) $\int_1^4 \sqrt{4+\frac{1}{t^2}+4t^2}\, dt =$

$\int_1^4 \sqrt{(2t+\frac{1}{t})^2}\, dt = \int_1^4 (2t+\frac{1}{t})\, dt$

$= (t^2+\ln(t))\Big|_1^4 = (16+\ln(4)) - (1+0)$

$= 15+\ln(4)$