

Difficulty guide for worksheet:

C-level or B-level exam problem: 1, 2, 4, 6

A-level exam problem or challenge for extra study: 5

beyond the scope and/or removed from syllabus: 3

1. Which of the following does *not* parametrize a line or some portion of a line? Explain your answer.

(a) $\mathbf{r}(t) = \langle 2 + 3t, 9 - t, 12 + 7t \rangle$

(c) $\mathbf{r}(t) = \langle 2 \cos(2t), 5 + 3 \cos(2t), \sin(2t) \rangle$

(b) $\mathbf{r}(t) = \langle 1 - t^2, 3 + 3t^2, t^3 \rangle$

(d) $\mathbf{r}(t) = \langle t^3, 4 - 8t^3, 8 + 3t^3 \rangle$

2. Find a parametrization of each described curve.

(a) A circle of radius 3 with center $(-2, 3, 1)$, lying in a plane parallel to the xz -plane.(b) The ellipse $4y^2 + 9z^2 = 36$ translated to have center $(-1, 10, -5)$.(c) The intersection of the surfaces $y^2 - z^2 = x - 2$ and $y^2 + z^2 = 9$.(d) The intersection of the sphere $x^2 + y^2 + z^2 = 1$ and the paraboloid $z = x^2 + y^2$.

3. We will show that the curve with the following parametrization lies in a plane.

$$\mathbf{r}(t) = \langle t^2 - 1, t - 2t^2, 4 - 6t \rangle$$

(a) Show that the points on the curve at $t = 0$, $t = 1$, and $t = 2$ do not lie on a line. Then find an equation of the plane that they determine.(b) Show that for all t , the points on \mathcal{C} satisfy the equation of the plane from part (a).

4. Find a parametrization of the line tangent to the curve at the indicated value of t .

(a) $\mathbf{r}(t) = \langle 1 - t^2, 5t, 2t^3 \rangle$ at $t = 2$

(b) $\mathbf{r}(t) = 4t^{-1}\mathbf{i} - 6t^{-3}\mathbf{k}$ at $t = 1$

5. For $0 \leq t \leq 4\pi$, the path of a particle is parametrized by

$$\mathbf{r}(t) = \langle \cos(t) \sin(t), \sin(t)^2, \sin(t) \rangle$$

(a) Show that the path of the particle is a closed loop.

(b) Let \mathcal{C} be the *curve* on which the particle travels. How many times does the particle traverse \mathcal{C} from $t = 0$ to $t = 4\pi$? Justify your answer.(c) Find an integral whose value is the length of \mathcal{C} .

(d) Find an integral whose value is the distance traveled by the particle.

6. Calculate the length of the described curve.

(a) $\mathbf{r}(t) = \langle 4t, \sqrt{3}t^{3/2}, t^{3/2} \rangle, 0 \leq t \leq 1$

(b) $\mathbf{r}(t) = \langle 2t, \ln(t), t^2 \rangle, 1 \leq t \leq 4$