Calculus 251:C3 Worksheet 12.1-12.2

(1) For each pair P, Q, find the components of $\overrightarrow{PQ}$ and calculate $\left\|\overrightarrow{PQ}\right\|$

(a) $P = (-3, -5)$, $Q = (4, -6)$
$\overrightarrow{PQ} = \langle 4 - (-3), -6 - (-5)\rangle = \langle 7, -1\rangle$
$\left\|\overrightarrow{PQ}\right\| = \sqrt{7^2 + (-1)^2} = \sqrt{50} = 5\sqrt{2}$

(b) $P = (2e, 1 - 2\pi)$, $Q = (2e + \pi, 1 + \pi)$
$\overrightarrow{PQ} = \langle (2e + \pi) - (2e), (1 + \pi) - (1 - 2\pi)\rangle = \langle \pi, 3\pi\rangle$
$\left\|\overrightarrow{PQ}\right\| = \sqrt{(\pi)^2 + (3\pi)^2} = \sqrt{10\pi^2} = \pi\sqrt{10}$

(c) $P = (3, -8, 2)$, $Q = (7, 4, -7)$
$\overrightarrow{PQ} = \langle 7 - 4, 4 - (-8), -7 - 2\rangle = \langle 3, 12, -9\rangle$
$\left\|\overrightarrow{PQ}\right\| = \sqrt{3^2 + 12^2 + (-9)^2} = \sqrt{234} = 3\sqrt{26}$

(d) $P = (1, 2, 3, 4)$, $Q = (3, -1, 5, -1)$ [*Note:* Yes, this problem is in $\mathbb{R}^4$]
$\overrightarrow{PQ} = \langle 3 - 1, -1 - 2, 5 - 3, -1 - 4\rangle = \langle 2, -3, 2, -5\rangle$
$\left\|\overrightarrow{PQ}\right\| = \sqrt{2^2 + (-3)^2 + 2^2 + (-5)^2} = \sqrt{42}$

(2) Perform the indicated vector operation.

(a) $\langle -4, 6\rangle - \langle 2, -3\rangle = \langle -6, 9\rangle$

(b) $\langle 3, 8, \pi\rangle + 2\langle 2, -4, -2\pi\rangle = \langle 7, 0, -3\pi\rangle$

(c) $2(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) - 3(\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) = 3\hat{\mathbf{i}} - 13\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

(d) $\langle \sin^2(\frac{\pi}{7}), \ln 27, \sqrt{2}\rangle - \langle -\cos^2(\frac{\pi}{7}), \ln 9, \sqrt{3}\rangle = \langle 1, \ln 3, \sqrt{2} - \sqrt{3}\rangle$

(3) Find the unit vector $\vec{e}_{\vec{v}}$ where $\vec{v} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$
$\vec{e}_{\vec{v}} = \dfrac{2}{\sqrt{13}}\hat{\mathbf{i}} - \dfrac{3}{\sqrt{13}}\hat{\mathbf{j}}$

(4) Find the vector $\vec{v}$ which satisfies the equation $3\vec{v} - \langle 3, 2, -5\rangle = \langle 0, 1, 2\rangle$
Let $\vec{v} = \langle a, b, c\rangle$. Then we have

$$\begin{aligned}
3\langle a, b, c\rangle - \langle 3, 2, -5\rangle &= \langle 0, 1, 2\rangle \\
\langle 3a, 3b, 3c\rangle - \langle 3, 2, -5\rangle &= \langle 0, 1, 2\rangle \\
\langle 3a - 3, 3b - 2, 3c + 5\rangle &= \langle 0, 1, 2\rangle
\end{aligned}$$

Now, $3a - 3 = 0 \Rightarrow a = 1$, $3b - 2 = 1 \Rightarrow b = 1$, and $3c + 5 = 2 \Rightarrow c = -1$.
So $\vec{v} = \langle 1, 1, -1\rangle$

(5) Let $\vec{u} = \langle 1, 3\rangle$, $\vec{v} = \langle 1, -1\rangle$, and $\vec{w} = \langle 3, 1\rangle$. Write $\vec{u}$ as a linear combination of $\vec{v}$ and $\vec{w}$.

Let $s, t \in \mathbb{R}$. We need to write $\vec{u} = s\vec{v} + t\vec{w}$, so

$$\langle 1,3\rangle = s\langle 1,-1\rangle + t\langle 3,1\rangle$$
$$\langle 1,3\rangle = \langle s,-s\rangle + \langle 3t,t\rangle$$
$$\langle 1,3\rangle = \langle s+3t, -s+t\rangle$$

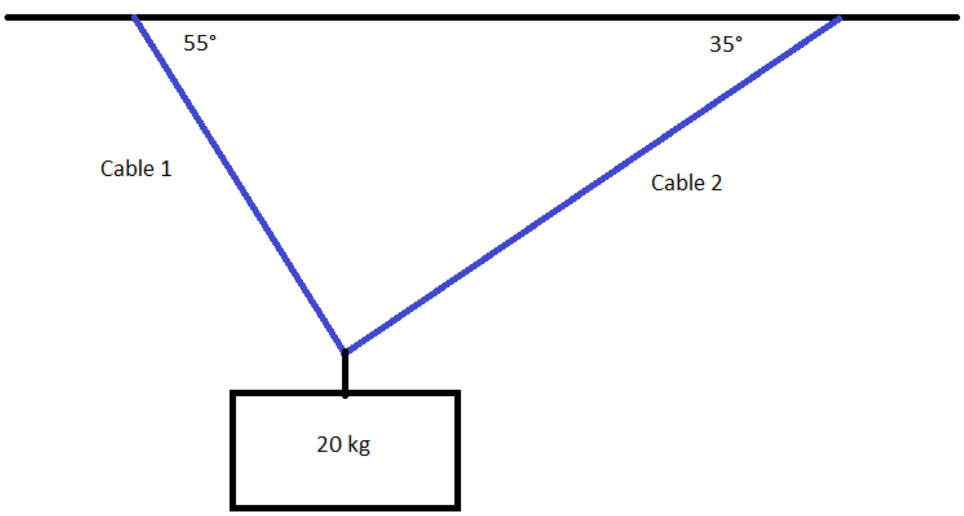
So we just need to solve the system of equations $s + 3t = 1$ and $-s + t = 3$. Adding the equations yields $4t = 4$, so $t = 1$. Substitute to find $s = -2$. Therefore we have $\vec{u} = -2\vec{v} + \vec{w}$.

(6) Find a parameterization of the line through $P = (0, 2, 4)$ and $Q = (5, -3, 3)$.

$\overrightarrow{PQ} = \langle 5, -5, -1\rangle$, so $\vec{v} = \langle 5, -5, -1\rangle$ is a direction vector for the line. We use $\vec{x}_0 = \overrightarrow{OP} = \langle 0, 2, 4\rangle$. We can now parameterize the line as

$\vec{r}(t) = \vec{x}_0 + t\vec{v} = \langle 0, 2, 4\rangle + t\langle 5, -5, -1\rangle = \langle 5t, 2 - 5t, 4 - t\rangle$

(7) Find the magnitudes of the forces on cables 1 and 2 in the following diagram:



Three forces act at the point where the mass is suspended, $\vec{F}_1$ along cable 1, $\vec{F}_2$ along cable 2, and $f_g = \left\|\vec{F}_g\right\| = (20kg)(-9.8m/s^2) = -196N$ which is the force of gravity acting on the mass in the downward direction. Since the block has constant position, its acceleration must be zero and therefore the net force at the point of intersection is zero.

Next, we need to write the forces on the cables in component form.

Let $f_1 = \left\|\vec{F}_1\right\|$ and $f_2 = \left\|\vec{F}_2\right\|$. Using right triangle trigonometry and the fact that $\vec{F}_1$ is acting up and to the left and $\vec{F}_2$ is acting up and to the right, we have

$$\vec{F}_1 = f_1\langle -\cos 55°, \sin 55°\rangle$$

$$\vec{F}_2 = f_2\langle \cos 35°, \sin 35°\rangle$$

$$\vec{F}_g = \langle 0, -196N \rangle$$

Since the net force is 0, we must have $\vec{F}_1 + \vec{F}_2 + \vec{F}_g = \langle 0, 0 \rangle$, which gives us the system of equations

$$-\cos 55^\circ f_1 + \cos 35^\circ f_2 = 0$$

$$\sin 55^\circ f_1 + \sin 35^\circ f_2 - 196 = 0$$

Solving the first equation for f_1 gives us $f_1 = \frac{\cos 35^\circ}{\cos 55^\circ} f_2$. Substituting into the second equation gives us

$$\sin 55^\circ \left(\frac{\cos 35^\circ}{\cos 55^\circ} f_2 \right) + \sin 35^\circ f_2 = 196$$

$$f_2 \left(\frac{\sin 55^\circ \cos 35^\circ}{\cos 55^\circ} + \sin 35^\circ \right) = 196$$

$$f_2 = \frac{196}{\frac{\sin 55^\circ \cos 35^\circ}{\cos 55^\circ} + \sin 35^\circ} \approx 112.42N$$

And then $f_1 = \frac{\cos 35^\circ}{\cos 55^\circ} f_2 \approx 160.55N$