Calculus 251:C3 Reading Guide - 6/10/2020

Section 14.7 Optimization in Several Variables

This section is essentially the multivariable version of finding relative extrema using derivatives. As you are reading through it, you should always be thinking, "What did this look like in Calculus I? What is different now?" I will also mention that the title of the section is a bit misleading, since the book only considers functions of exactly two variables $z = f(x, y)$. It is possible to extend these techniques to functions of three or more variables, but that is beyond the scope of this course. If you are really curious (and I mean *really* curious), you could look at this PDF if you want a basic introduction of what happens with more variables. Click at your own risk.

Find the following definitions/concepts/formulas/theorems:

- local extreme values
- critical point
- Fermat's Theorem
- saddle point
- discriminant (a/k/a Hessian determinant)
- Theorem: Second Derivative Test (for functions of exactly two variables)
- global extrema (note the book's usage of this term is strange because it is talking about a domain $\mathcal{D}$ which is not all of $\mathbb{R}^2$)
- bounded
- interior point
- boundary point
- Theorem: Existence and Location of Global Extrema

The proof of Fermat's Theorem is really just applying the one-variable version to both partials. Not too bad. It's more important for you to know the theorem than its proof.

The proof of the second derivative test is at the end of the section and includes the defintion of a quadratic form. Again, if you plan to be a math major (or are really curious), you should spend some time considering it. If neither of those applies to you, you can skip it.

Examples 1 and 2 both involve finding a relative maximum or minumum if you already have the graph of a function. The function for example 2 would be extremely annoying to do by hand, so they find the partials and the critical point with computer software. In real-world engineering applications, perople often use a CAS to do this part of the work, but they still have to understand multivariable calculus well enough to know what the answers mean and to sanity check them.

Examples 3, 4, and 5 are really the typical problems for this topic. You are given a function. You compute partials, find critical points, compute second-order partials, find the discriminant, use it to classify the critical points. Example 5 is definitely tougher than the others because the second derivative test fails and you have to look at traces of planes through the critical point.

The Global Extrema section really says that if you are looking at a domain $\mathcal{D}$ which is a closed subset of $\mathbb{R}^2$, then you always have extreme values and they always happen at the critical points or on the boundary. This was easier in Calc I, because the closed subset of the domain was an interval $[a, b]$ and you checked function values at the critical points and the endpoints a and b. Your maximum and minumum were just the biggest and smallest function values. The reason this is more complicated here is that the boundary isn't just two points, it's infinitely many points. So you need to divide your boundary into pieces, figure out what the function looks like when restricted to that part of the boundary, and then use single variable calculus to find the minimum and maximum on each piece.

Example 6 isn't bad because $\mathcal{D}$ is a rectangle with sides parallel to the coordinate axes, which makes the calculations on the boundary easy. You should definitely make sure you can do a question similar to this one. Example 7 is a little worse because the boundary is circular so you have to parametrize it with two semicircles. Example 8 is annoying and cool at the same time, since you have to figure out what the domain is based on the physical situation.