**Calculus 251:C3 Reading Guide - 6/4/2020**

**Section 14.5 The Gradient and Directional Derivatives**

NOTE1: This material is NOT going to be on midterm 1!

NOTE 2: We are finally at the part of the course where we only do one section of the book per day.

In this section, we are going to learn about the gradient, one of the most useful concepts in all of multivariable calculus. By definition, it looks like we are just giving a name to the vector whose components are the partial derivatives of a function. But this vector has some amazing properties which will allow use to find directional derivatives, paths of steepest ascent, normal vectors to level curves and surfaces. The gradient will also be a critical tool that we use to analyze vector fields in chapter 16.

Find the following definitions/concepts/formulas/theorems:

- gradient
- Theorem: properties of the gradient
- Theorem: Chain rule for paths
- directional derivative (definition and formula for finding it)
- Theorem: interpretation of the gradient (ask yourself why the second bullet point has to follow from the first)
- Theorem: gradient as a normal vector

Examples 1 and 2 are direct calculations using the definition of the gradient.

If you are curious, you should try proving one or more of the properties of the gradient (Theorem 1).

Example 3 is an application of the chain rule for gradients, and example 4 is an application of the chain rule for paths. These are still fairly straightforward.

Examples 5 and 6 are computations of directional derivatives. If you believe that the formula is correct (it is), these are not bad. The really important thing to notice is that we *must* use a unit vector when we calculate directional derivatives. If you leave out the normalization (i.e. changing the length to 1) step, you will have no hope of getting a right answer. I want you to have hope, so please remember.

The Graphical Insight on page 834 and examples 7-10 are all intended to get you excited about how cool the gradient is. Hopefully, they will work on you. This is another one of those times where you may need to think about these examples several times over a period of days while sipping a hot beverage. Multivariable calculus has a lot of those...