Calculus 251:C3 Reading Guide - 5/28/2020

Section 12.5 Planes in 3-Space

In this section, we are going to take many of the concepts that you are (hopefully) very comfortable with regarding lines in $\mathbb{R}^2$ and extend them to planes in $\mathbb{R}^3$. Back in high school algebra, you learned that the standard form of the equation of a line is $ax + by = c$. We called this a linear equation because its graph in $\mathbb{R}^2$ is a line. But the concept of linear equations generalizes to any number of dimensions. In fact, any equation in which every variable appears with only a (possibly zero) coefficient is called a linear equation in any number of dimensions. Of course, in dimensions higher than two the graph is no longer a line. But recall the term "linear combination" as we applied it to vectors: this is the sense in which these equations are linear. These equations are all of the form "some linear combination of the variables=constant". In $\mathbb{R}^3$, every linear equation describes a plane.

Find the following definitions/formulas/theorems:

- normal vector (reminder from Calculus I: the normal line to a curve at a point is perpendicular to the tangent line at the same point)
- Geometric description of a plane
- Equations of a plane (3 different forms)
- When are two planes parallel?
- collinear points
- trace (of a plane)

Example 1 shows several different ways to find and write the equation of a plane given a normal vector and a point in the plane. You should try to become comfortable with all of them.

Examples 2 involves parallel planes. It really is as easy as it looks. No, you're not missing some deeper meaning.

Example 3 shows the method for finding an equation of the plane through any three non-collinear points. There are a lot of steps, but each step is pretty easy. The CAUTION at the bottom of the example is important, since it discusses one of the most frequent mistakes on this problem type. Don't use position vectors (like $\overrightarrow{OP}$) because they are not usually in the plane for which you are trying to find an equation! The most important thing to remember when finding your two vectors which span the plane: you need to use all three points. The other part of the caution doesn't acutally matter. What would happen if you used $\overrightarrow{PQ} \times \overrightarrow{QR}$?

Example 4 shows how to find the intersection of a plane and a line. What would it mean geometrically if the line and the plane did not intersect? What would happen during the process used in this example in that case, i.e. how would you be able to tell that there was no point of intersection?

Example 5 demonstrates how to graph a plane and to find the traces of the plane. The basic idea is that the parameterization of the x-axis is $\langle t, 0, 0 \rangle$, so setting $y = z = 0$ and plugging those in to the equation of the plane will give you the unique point which is both in the plane and on the x-axis. If you then repeat for the y- and z-axes, you have all three intercepts. The triangle with those three vertices lies in the plane, so that will allow you to graph the plane. What happens if you don't get a single point? Well, that will only happen if one or more of the coefficients is 0. You will then either get the equation $0 = 0$, in which case the entire axis is in the plane or you will get $0 = a$ for some nonzero a in which case the plane does not intersect that axis at all. In either case, you should be able to figure out what the graph looks like.